Contract NAS 1–5199 Report No. 3444

RESEARCH AND DEVELOPMENT OF THE VORTEX VALVE PRINCIPLE AND ITS APPLICATION TO A HOT GAS (5500°F) SECONDARY INJECTION THRUST VECTOR CONTROL SYSTEM

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SECTION 1

INTRODUCTION

This program is the study of a vortex valve controlled secondary injection thrust vector control system, operating with highly aluminized gas from a solid propellant gas generator (SPGG). Various performance characteristics will be determined , including static and dynamic system performance and the ability of the vortex valve to handle the aluminized hot gas. The application of this technique for thrust vector control of a solid propellant rocket engine, using direct engine bleed, will be considered.

SECTION 2

ACCOMPLISHMENTS THIS PERIOD

5500°F SITVC Single Vortex Valve System (For Test Firing Number 2)

Design parameters for the single vortex valve arrangement of the 5500°F SITVC System have been defined and incorporated in the design of the system components. A system layout drawing is complete and shown in Figure 1. Detail drawings were made of the individual parts, and the parts were procured or fabricated.

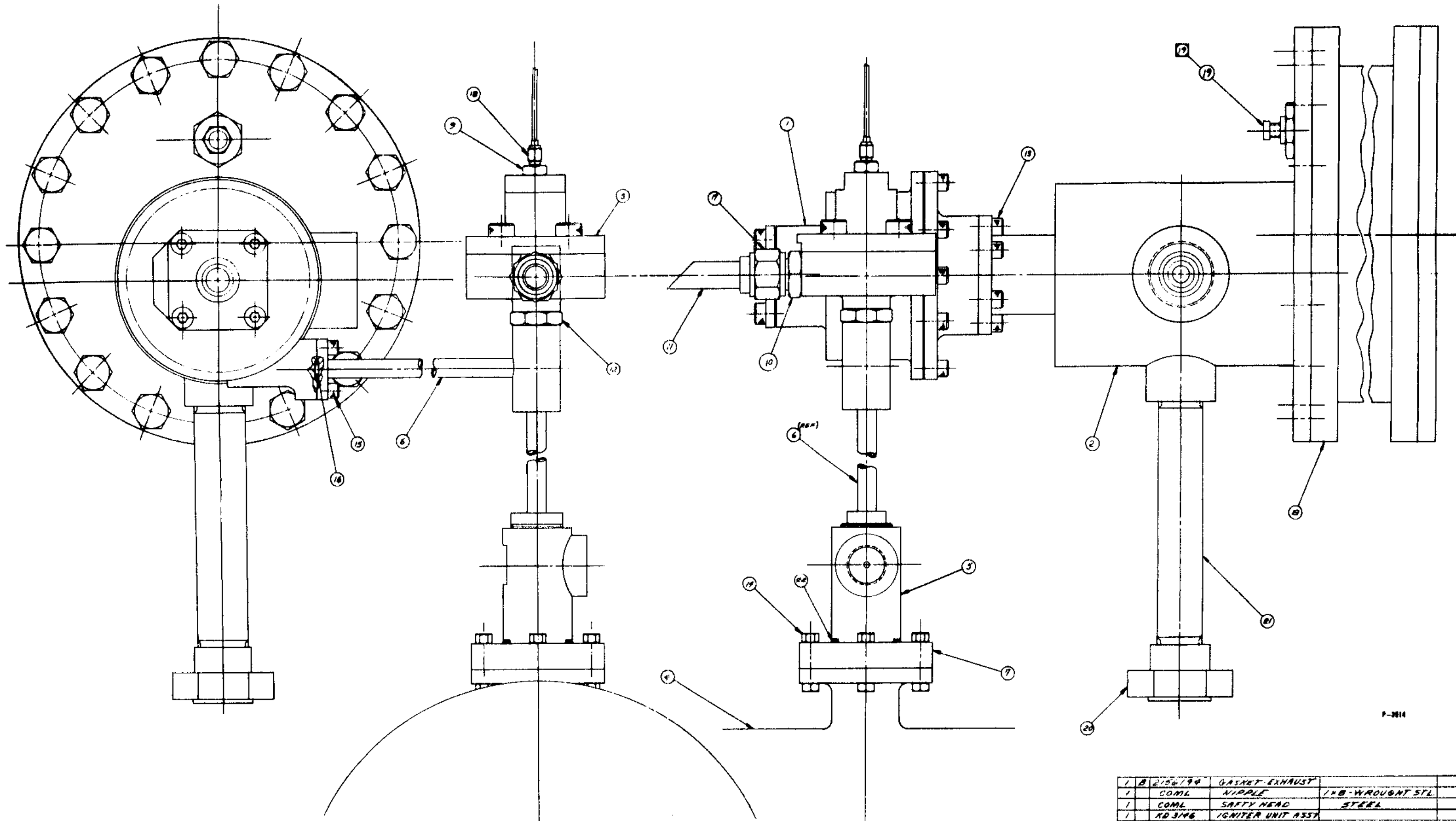
The single vortex valve system has been assembled and is now undergoing cold gas test. The cold gas test will provide the necessary adjustment check of the test programmer as well as the modulation characteristics of the system. A more complete description of this system will be incorporated in the next progress report. However, the test outline and procedure is presented in Appendix C of this report for information.

5500°F SITVC System (For Test Firing Number 3)

The layout drawing and all of the detail drawings for the two-vortex valve system have been completed. The system assembly is shown in Figure 2. Fabrication of the detail parts will be completed by 5 May 1966. Component test will be conducted as the subassemblies become available. Full scale system cold gas testing will not start until the single valve hot gas firing is completed. The single valve test should be accomplished by 15 May 1966.

2000°F Pilot Stage Test

The 2000°F pilot stage designed for the 5500°F SITVC System was flow-trimmed on cold gas and was not gas tested this period. The system as tested is shown in Figure 3. The cold gas testing consisted of flow trimming the pilot stage while it was mounted on the Phase I vortex power valves also shown in Figure 3. These valves are the same configuration as the 5500°F SITVC valves and therefore provide the same impedance loading on the pilot stage. The principal objective of the test was to demonstrate flow modulation and steady-state performance on

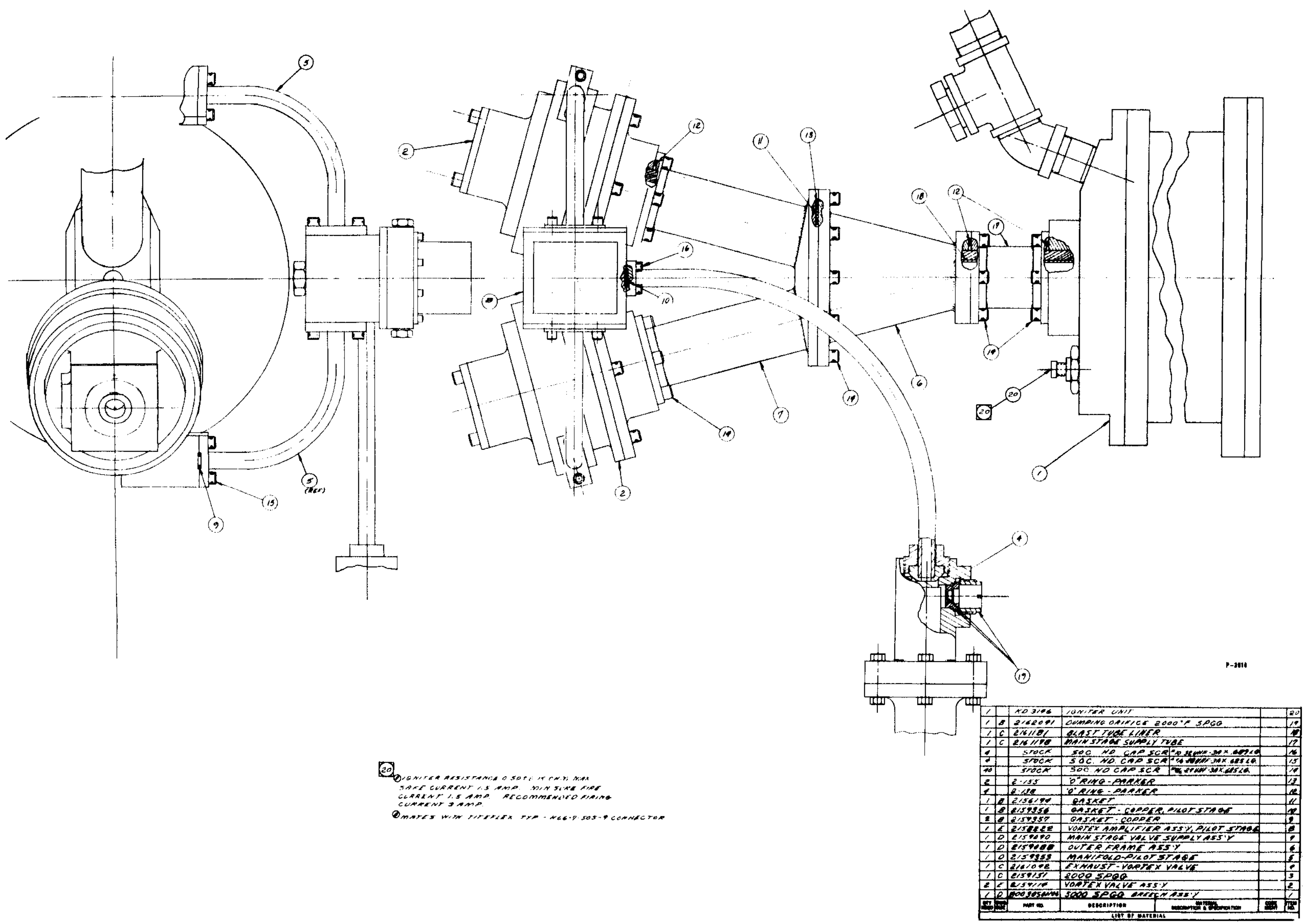


[19]
(a) IGNITER RESISTANCE 0.50±0.15 OHM MAX.
SAFE CURRENT 0.25 AMP MIN - SURE FIRE
CURRENT 1.5 AMP - RECOMMENED FIRING
CURRENT 3 AMP
(b) MATES WITH TITEFLEX TYPE NO6-9-5059 CONNECTOR

QTY REQD	DWG SIZE	PART NO.	DESCRIPTION	MATERIAL DESCRIPTION & SPECIFICATION	CODE IDENT	ITEM NO.
1	B	2156194	GASKET-EXHAUST			22
1		COML	NIPPLE	1×8-WROUGHT STL.		21
1		COML	SAFTY HEAD	STEEL		20
1		KD 3146	IGNITER UNIT ASSY			19
2		STOCK	NUT & SLEEVE-S.STL	AN818-8, AN819-8		18
1		STOCK	NUT & SLEEVE-S.STL	AN818-[illegible], AN819-[illegible]		17
1			GASKET-COPPER			16
4		STOCK	SOC.HD CAP SCR.	1/4-28 UNF-3A X .625 LG CRES		15
4		STOCK	HEX.HD.CAP SCR	1/4-28 UNF-2C X 1.75 LG.		14
6		STOCK	SOC. HD. CAP SCR.	[illegible] UNF-3A X .625 LG CRES.		13
1		STOCK	UNION	AN815-8-S.STL.		12
1		STOCK	TUBING	1/4 O.D.-S.STL.		11
1		STOCK	UNION	AN815-12-S.STL.		10
1		STOCK	FITTING-REDUCER	AN919-1-S STL		9
1	D	DM36501080	5500°F SPGG			8
1			TEST GENERATOR FLANGE			7
1	D	2161502	MANIFOLD-PILOT STAGE			6
1	C	2156199	EXHAUST-VORTEX VALVE			5
1	F	4542000	2000°F SPGG			4
1	D	2154636	MANUAL CONTROL VAVLE			3
1	E	2161505	5500°F SPGG FLANGE			2
1	E	2161503	VORTEX VALVE ASSY			1

LIST OF MATERIAL

Figure 1 - 5500°F SITVC Single Vortex Valve System (for Test Firing Number 2)



QTY REQD	DWG SIZE	PART NO.	DESCRIPTION	MATERIAL DESCRIPTION & SPECIFICATION	CODE IDENT	ITEM NO.
1		KD 3196	IGNITER UNIT			20
1	B	2162091	DUMPING ORIFICE 2000°F SPGG			19
1	C	2161181	BLAST TUBE LINER			18
1	C	2161178	MAIN STAGE SUPPLY TUBE			17
4		STOCK	SOC. HD. CAP SCR	#10-32UNF-3A X .687 LG		16
4		STOCK	SOC. HD. CAP SCR	1/4-28UNF-3A X .625 LG		15
40		STOCK	SOC. HD. CAP SCR	5/16-24UNF-3A X .625 LG		14
2		2-155	"O" RING - PARKER			13
4		2-138	"O" RING - PARKER			12
1	B	2156194	GASKET			11
1	B	2159356	GASKET - COPPER, PILOT STAGE			10
2	B	2159357	GASKET - COPPER			9
1	E	2158222	VORTEX AMPLIFIER ASS'Y, PILOT STAGE			8
1	D	2159090	MAIN STAGE VALVE SUPPLY ASS'Y			7
1	D	2159088	OUTER FRAME ASS'Y			6
1	D	2159353	MANIFOLD-PILOT STAGE			5
1	C	2161092	EXHAUST - VORTEX VALVE			4
1	C	2159151	2000 SPGG			3
2	E	2159114	VORTEX VALVE ASS'Y			2
1	D	800305MW	5000 SPGG BREECH ASS'Y			1

LIST OF MATERIAL

Figure 2 - 5500°F SITVC System (for Test Firing Number 3)

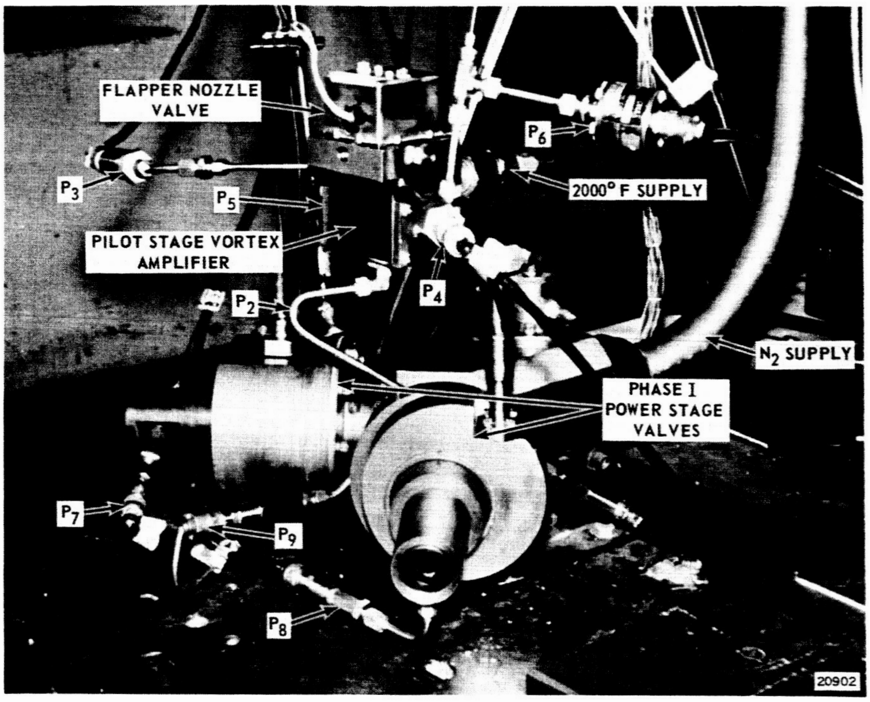


Figure 3 - 2000°F Hot and Cold Gas Pilot Stage Test

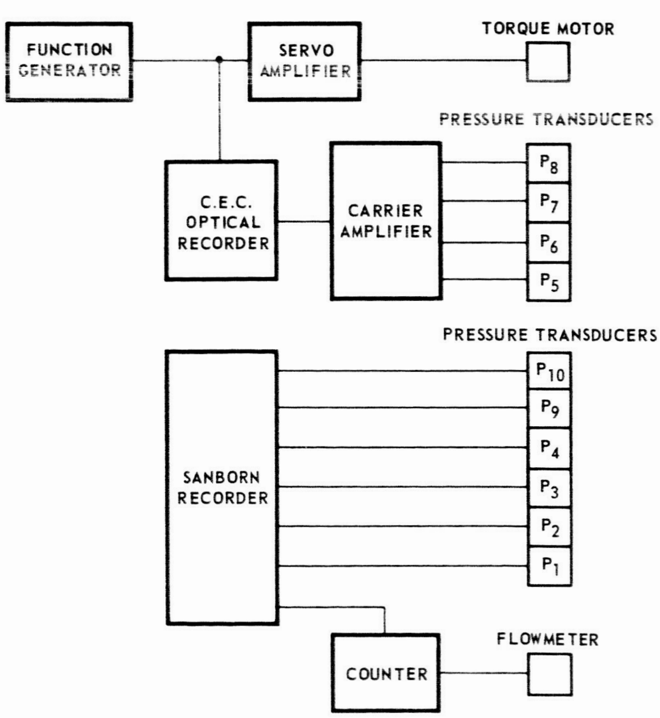


Figure 4 - Instrumentation Schematic for 2000°F Pilot Stage Test

both cold and hot gas. The system demonstrated a flow modulation of 4.25:1 on cold gas and 1.66:1 on hot gas. However, the control-to-supply pressure ratio was considerably above the normal operating range, which was responsible for the apparent reduction in flow modulation.

The system was instrumented in keeping with the test objective. Figure 4 is the instrumentation schematic, and the various pieces of equipment used are itemized below. The pilot stage input and output and the main stage output were recorded by the optical recorder while other parameters were recorded on the Sanborn Recorder.

<u>Recorder</u>

Sanborn - Model 858-5460
C.E.C. - Type 5-124

<u>Servo Amplifier</u>

Bendix Design

<u>Function Generator</u>

Hewlett Packard Model 202A

<u>Pressure Tranducers</u>

Dynisco PT49A
Dynisco PT135
Dynisco PT31

<u>Flowmeter</u>

Cox Turbine GH32 and GH12

<u>Frequency Counter</u>

Cox Model 950

Cold Gas Test

For cold gas testing of the 2000°F Pilot Stage, the system was assembled as shown in Figure 3. The pilot and main stage were supplied by independent regulated pressure sources, so that the optimum pressure level and optimum control-to-supply pressure ratio could be determined. Pressures and flows were measured in various locations, as shown in Figure 5, to provide the necessary information for impedance matching of the pilot stage. The pressure distribution in the system is summarized

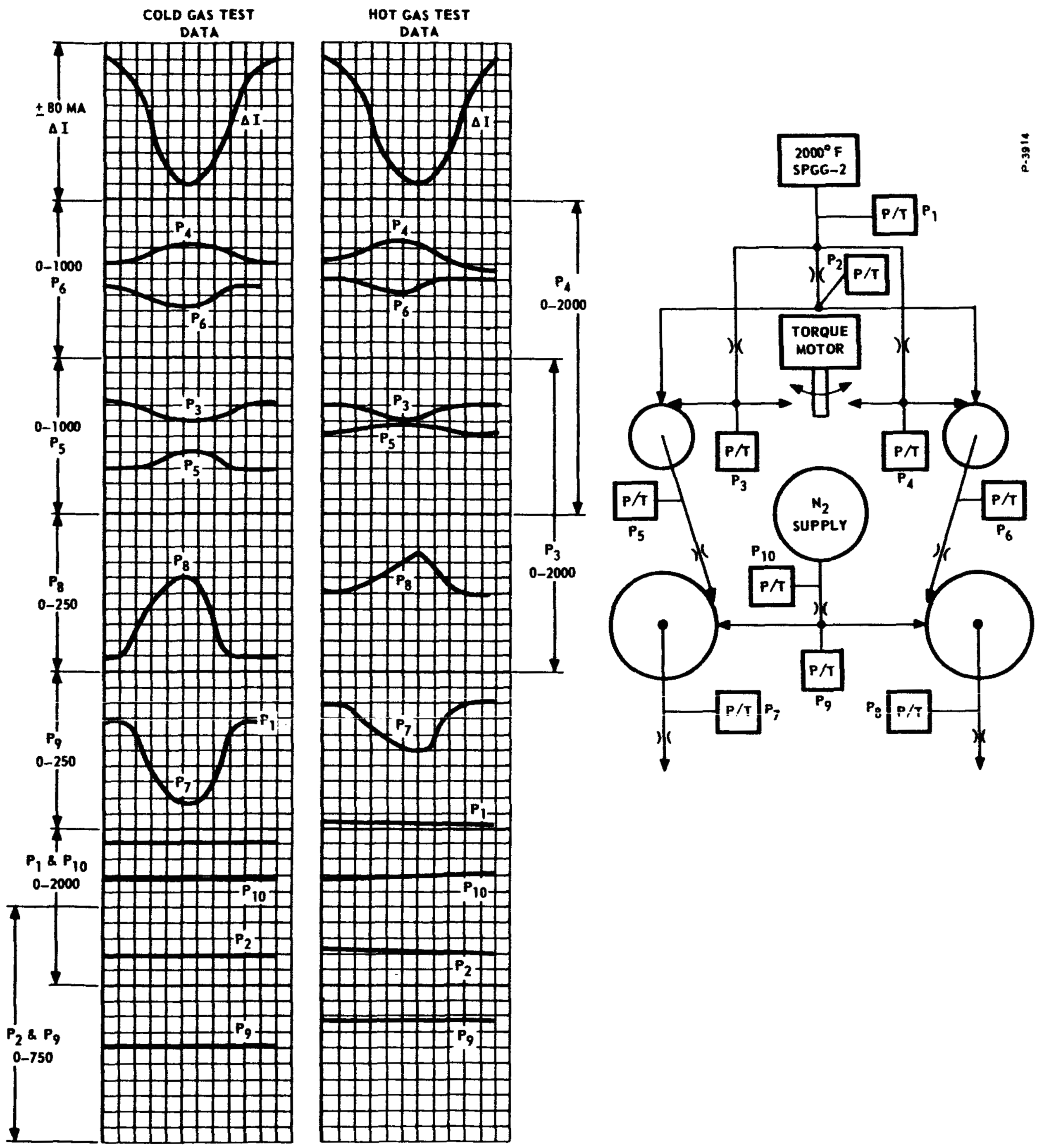


Figure 5 - Hot and Cold Gas Test Results for the 2000°F Pilot Stage

in Figure 5 for one complete modulation cycle. To produce these results, the system was cycled at a rate of 0.5 cycle per second.

The system demonstrated a flow modulation range of 4.25:1 with a maximum output flow of 3.56 lb/sec of nitrogen gas or the equivalent of 1.08 lb/sec of PJS, the 5500°F solid propellant gas. Referring to Figure 5, the output indicates saturation caused by the slight unbalance in the pilot stage output and by the over-driving of the main stage by the pilot stage. Taking the average low and high control-to-supply pressure ratio, a change from 1.02:1 to 1.37:1 provided a flow modulation of 4.25:1. This control-to-supply pressure ratio is slightly more than required, based on previous cold gas component tests. In the component tests, both vortex power valves were fully modulated with a control-to-supply pressure ratio of 1.26:1. The P_c and P_o pressure traces in Figure 5 are indicative of the saturation effect.

It was noted during cold gas testing that the modulation capabilities of the system are sensitive to the pressure ratio between the pilot stage and the main stage. This sensitivity is due to the high flow gain in the pilot stage. The most direct corrective action is to reduce the nozzle diameter, thereby increasing the flapper stroke. This will be accomplished before the next system test. It will require additional cold gas testing for verification but will not required another pilot stage hot gas test.

Hot Gas Test

For the pilot stage hot gas test, the pilot stage cold gas supply tube was removed and the 2000°F SPGG was attached. The 2000°F SPGG was ignited before flowing the main stage. This sequence of events was chosen to prevent back-pressurizing the 2000°F SPGG and causing a possible malfunction. The Phase I main stage supply valves were loaded in the same manner as during the cold gas test. For some undetermined reason, the main stage supply pressure did not reach operating pressure until approximately 19 seconds after ignition of the SPGG. During this time, the torque motor signal was held in the null position. During this 19-second interval, the pilot stage supply pressure was decreasing at 30 psi per second and the main stage supply pressure was increasing at 70 psi per second. Inspection of the pressure traces while the test was in progress indicated the chain of events which were taking place. A signal was immediately applied to the torque motor, and the main stage supply pressure before burn-out. The reduction of the main stage supply pressure was partially successful since the pressure traces indicated that the system was being modulated by the pilot stage before burn-out.

The reduced hot gas test data shown in Figure 5 is taken at 20 seconds after ignition of the 2000°F SPGG. It is interesting to note the adverse effect of control-to-supply pressure ratio on output modulation. This ratio changed from an average low of 1.21:1 to a high 1.37:1 (instead of from a desired 1.0:1 to 1.37:1). This limited control modulation accounts for the low output modulation of 1.66:1 on the main stage. By comparison, the cold gas test output modulation was 4.25:1, and it can be concluded that the hot gas performance would have been similar if a sufficiently higher pilot stage supply pressure had been achieved during the hot gas tests.

The 2000°F SPGG ballistic data is presented in Figure 6. Integrating the area under the ballistic trace yields an average supply pressure of 2115 psia. This pressure will cause the grain to burn at an average rate of 0.1173 inch per second and deliver a flow rate of 0.486 lb/sec. The average grain weight is 16.1 pounds, and burn-out occurred at 33 seconds; this data yields an average flow rate of 0.488 lb/sec. This correlation indicates that the regressive burning (drop-off of supply pressure) is inherent in the SPGG and is not due to system leakage.

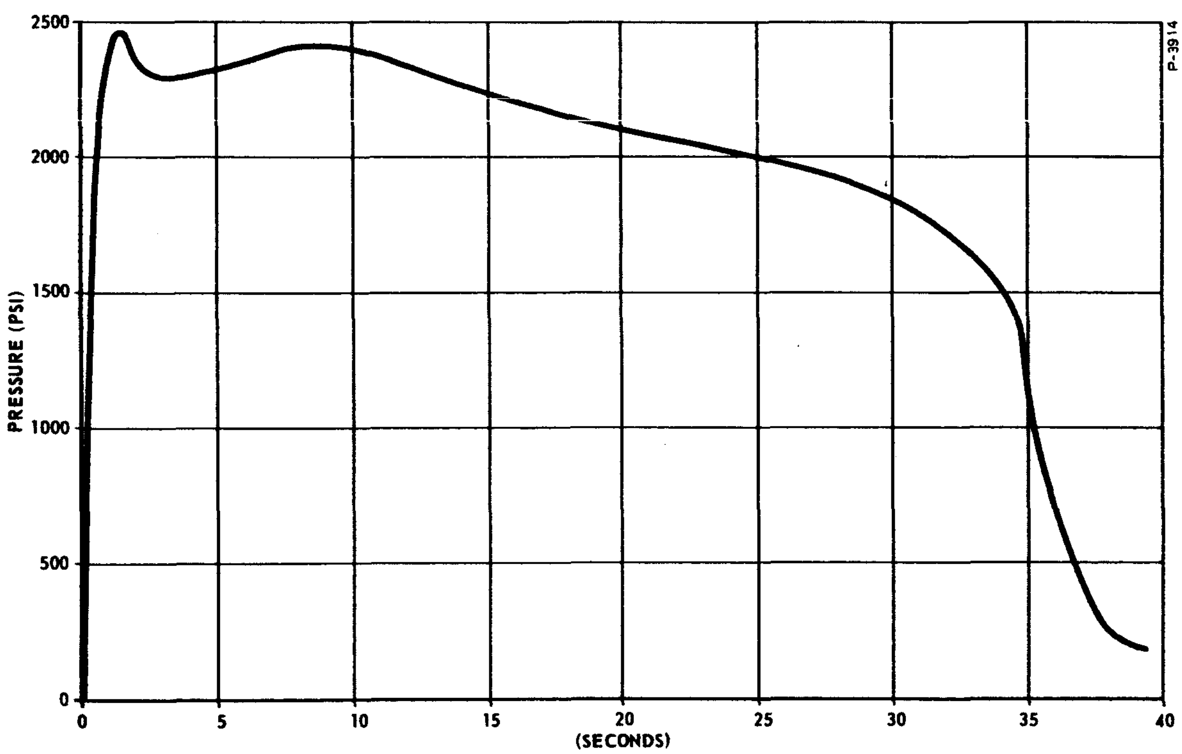


Figure 6 - 2000°F Hot Gas Pilot Stage Test (SPGG Ballistic Data)

What may be concluded from the ballistic data and system test results is that a pressure regulating device for the 2000°F SPGG is highly desirable if optimum performance is to be realized for a full-duration firing. A regulator valve which would satisfy the system requirement is discussed later in this report.

Post Firing Examination

The general condition of the pilot stage after the hot gas test was very good. None of the load orifices were clogged; nor was erosion apparent. There was only very minor leakage in the pilot stage which was around one of the Natorque seals on the flapper nozzle valve. One of the main stage vortex valves developed a minor leak at the injection ring. The copper seals held up very well with no leakage.

Thermal Analysis of Preheat Effects on a Vortex Valve

A thermal analysis was conducted on the 5500°F vortex valve button and chamber to determine the effectiveness of 2000°F control gas on preheating of these internal parts. The analysis is qualitive, since the values for the coefficient of heat transfer can only be estimated. The results appear to be realistic, based on past experience. The analysis shows that ignition of the 2000°F SPGG should take place some 10 seconds prior to igniting the 5500°F SPGG if the full heating benefit of the control flow is to be realized. This would allow approximately 20 seconds over-lap time when both generators are burning, from which only 15 to 18 seconds could be utilized for modulating the 5500°F vortex valve. Therefore, the ignition time of the 2000°F SPGG was selected to be 5 seconds prior to ignition of the 5500°F SPGG. This allows the parts to reach approximately 80 percent of the temperature they would attain in 10 seconds. The mathematical model and the analytical results are discussed in more detail in Appendix A.

Stability Analysis of an SPGG Pressure Regulating Valve

The recent pilot stage hot gas test emphasized the need for maintaining pressure regulation between the 2000°F SPGG and the 5500°F SPGG if optimum performance is to be realized. The requirement for a pressure regulation technique for the 2000°F SPGG seems inevitable since the three different hot gas tests on the 2000°F SPGG have shown severe regressive burn characteristics. The exact cause of the change in ballistic performance when a "half" charge load is used has not been determined.

It has been demonstrated earlier in the program that maximum output flow modulation can be achieved over a wide pressure range with a particular valve geometry, provided the control-to-supply pressure ratio is held within a reasonable limit. Regulation of the supply and control pressure ratio becomes more stringent as the differential control pressure range required to modulate the valve decreases. Since in the present system the differential control pressure is low, the effect on performance is more drastic when adverse changes occur in either the control or supply pressure.

The detailed analysis of a pressure regulating technique is given in Appendix B, along with a summary of the critical design parameters. This analysis was performed to determine if the balancing valve concept is stable and could be used to eliminate the pressure variation effects in the two hot gas generators. Figure 7 is a functional schematic showing

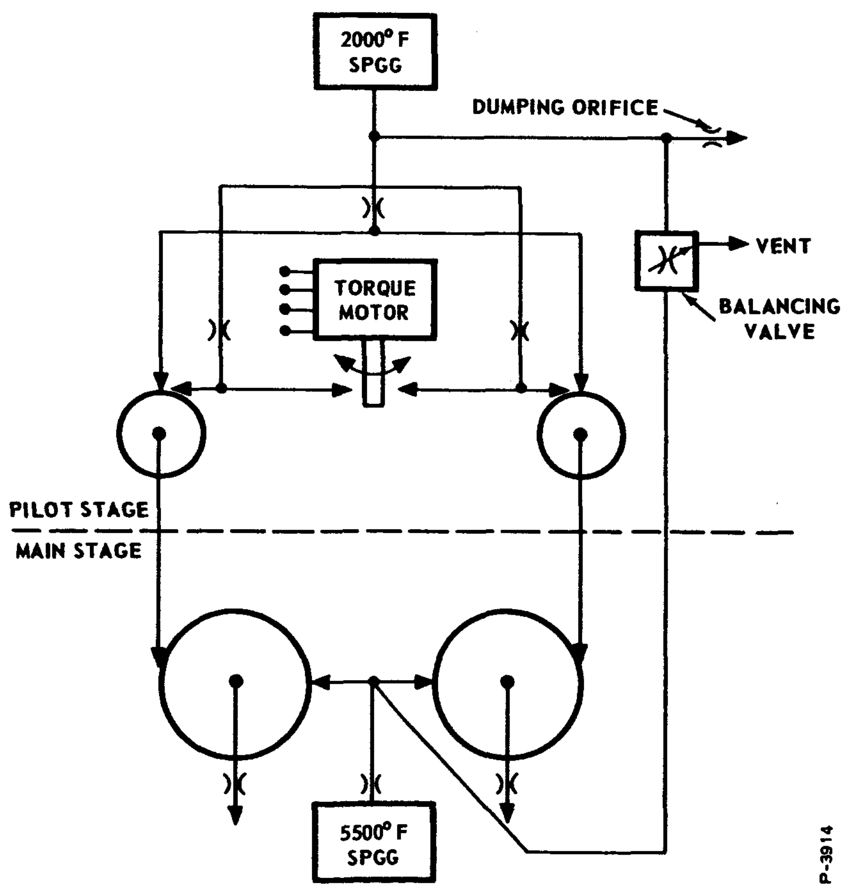


Figure 7 - 5500°F SITVC System Modification Schematic

how the balancing valve could be incorporated into the present hot gas system between the two stages. Note that the sensing point on the main stage is downstream of the 5500°F SPGG load orifice while the other sensing point is located at the 2000°F SPGG. This arrangement will insure system regulation regardless of the performance of either generator.

Essentially, the balancing valve is of simple relief-type construction with a damping chamber added for stability. The free-floating piston with a differential area is being acted on by the 2000°F SPGG and by the main stage supply pressure. A sealed chamber filled with liquid on the main stage side provides damping and at the same time deadheads the 5500°F hot gas, thus preventing an overboard leakage path. The other end of the piston provides a variable dumping orifice for the pilot stage. Any change in displacement of the piston, resulting from an unbalanced force due to a differential pressure, results in a change in the dumping area until the balance in forces has been re-established. A limit stop on the piston can be incorporated in the design to allow starting the 2000°F SPGG against a fixed impedance load, thus setting a minimum control pressure for system start.

Displacement of the piston forces the viscous liquid from one chamber to another through a restricting orifice thereby controlling the displacement rate. The damping can be increased or decreased by changing the viscosity of the fluid or the area of the restricting orifice.

The material requirements for the major parts of the valve are summarized in Table 1, along with appropriate design remarks.

Table 1 - Material Requirements for Pressure Balancing Valve

Part	Material	Remarks
Body	Stellite 6B	Full Annealed and Oxidized
Seat	Moly 1/2 Ti	Adquate for 2000°F Application
Piston Body	Aluminum Oxide	Low Coefficient of Thermal Expansion Low Density
Seat Mating Surface on Piston	Moly 1/2 Ti	Adequate for 2000°F Application
Piston Seals	Viton O-Ring	Good for Continuous Service at 500°F
O-Ring Slippers	Teflon	Low Coefficient of Friction

SECTION 3

PROBLEM AREAS

The following are possible major technical problem areas:

(1) Maintaining the proper pressure ratio between the main stage and the pilot stage. (A solution to this problem has already been conceived.)

(2) Short burning time of the 2000°F SPGG. (The grain supplier has been contacted and has proposed a solution which may be incorporated at a later date as the development testing dictates.)

SECTION 4

MEETINGS AND CONTACTS

Mr. G. L. Smith of NASA-Langley visited Bendix for a general review of the SITVC development program. The test schedule and program cost were discussed. Mr. Smith also witnessed the 2000°F pilot stage test during his visit.

Mr. J. G. Rivard visited NASA-Langley to discuss the SITVC system firing at ABL. The results of the visit were a definition of a work statement and the schedule for the SITVC system firing at ABL.

SECTION 5

PLANS FOR NEXT PERIOD

The next report period, which ends 2 June 1966, will include a hot gas firing and will indicate the flow modulation characteristics of the vortex valve in a SITVC system.

The following tasks will be completed during the next reporting period:

(1) A single vortex valve circuit will be set up and tested with 2000°F gas in the pilot stage and 5500°F in the power stage.

(2) A simulated single-axis SITVC system will be set up and tested for steady-state performance on cold gas.

(3) The test effort will be reviewed with respect to the next scheduled hot gas test.

(4) Insulation and tungsten will be procured for the #3 and #4 firings.

SECTION 6

PROGRAM SCHEDULE

The existing program plan is shown in Figure 8 and indicates the various subtasks and the planned period of accomplishment.

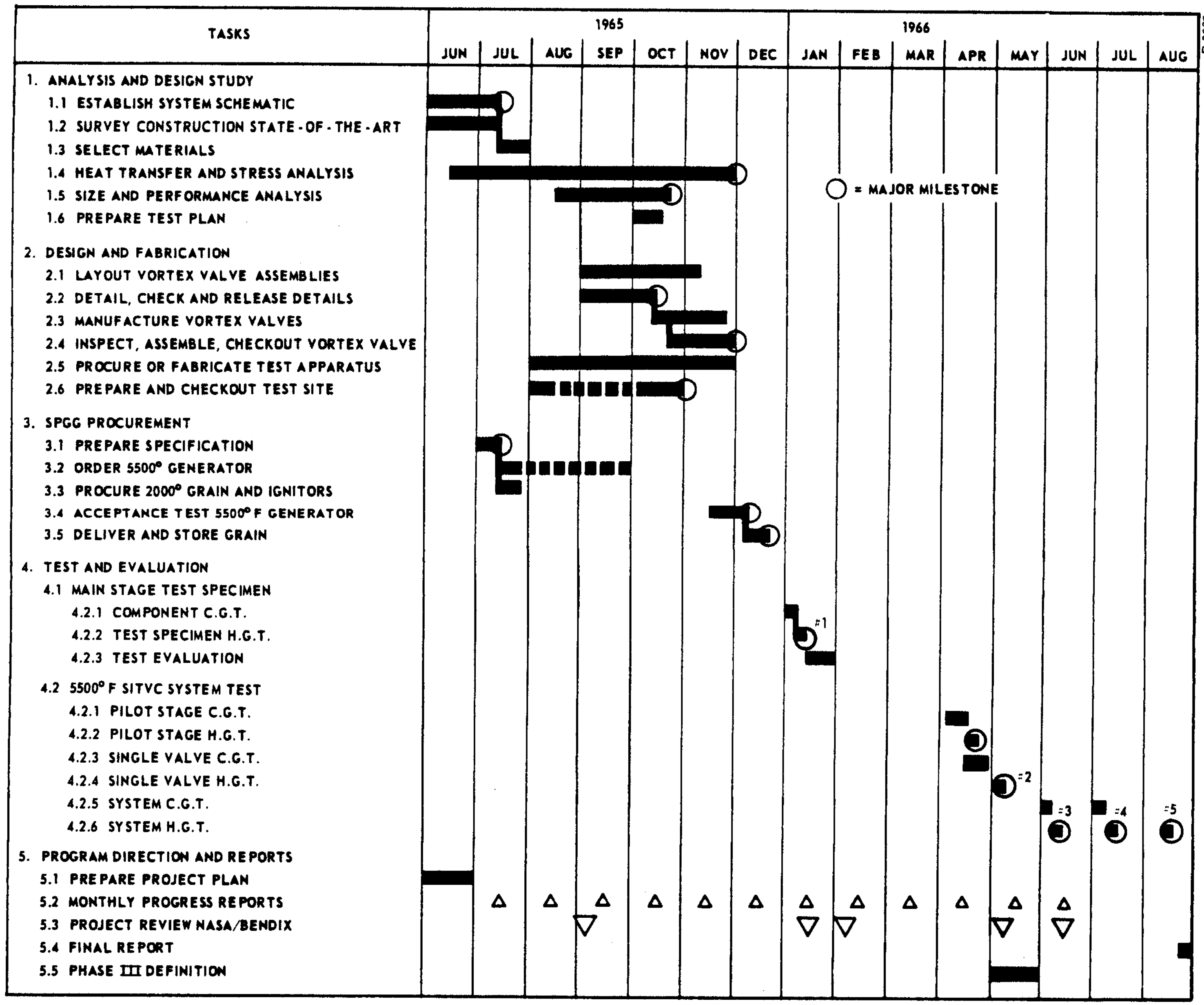


Figure 8 - Program Schedule

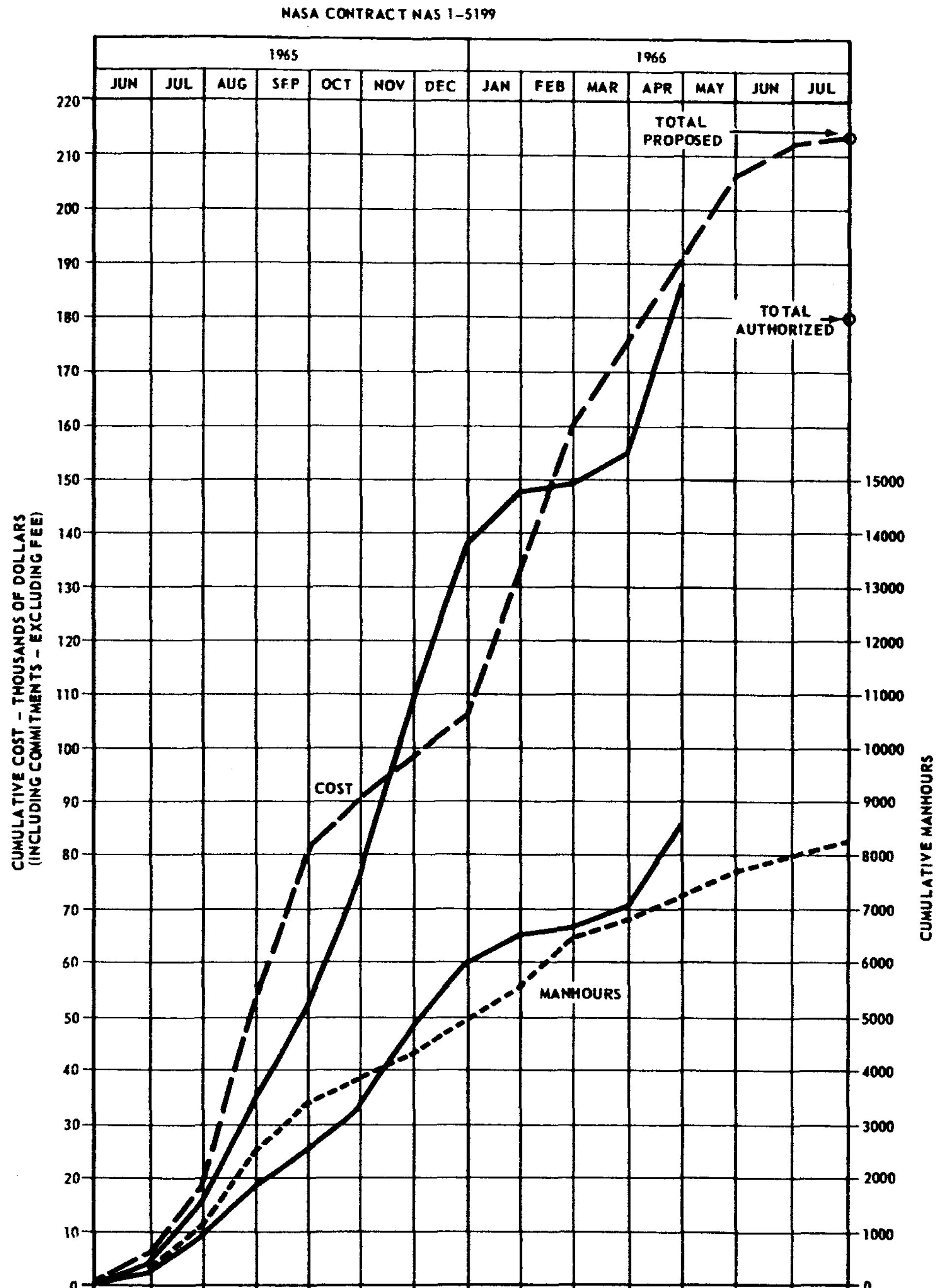
SECTION 7

MONTHLY FINANCIAL AND MANPOWER UTILIZATION REPORT

The cumulative manhour expenditures by category through April 30 are as follows:

Engineering	4642
Drafting	331
Technician	1610
Miscellaneous	770
Shop	1279

A graphic and tabular presentation of contract expenditures is shown in Figure 9.



	JUN	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN	JUL
FORECAST COST	5.08	18.26	53.21	81.23	90.76	97.82	106.28	115.74	160.1	175.0	190.7	206.6	210.9	214.0
ACTUAL COST (EXCLUDING COMMITMENTS)	4.79	15.09	29.47	48.06	60.38	90.46	116.52	134.85	136.7	156.1	180.6			
CURRENT COMMITMENTS		0.02	5.07	4.52	16.54	19.40	22.14	12.28	11.9	.2	4.6			
TOTAL COST (INCLUDING COMMITMENTS)	4.79	15.11	34.54	5258	76.92	109.88	138.66	147.13	148.6	156.3	185.2			
FORECAST MANHOURS	326	1224	2650	3465	3818	4364	4989	5644	6450	6803	7223	7660	7979	8223
ACTUAL MANHOURS	289	946	1805	2569	3311	4792	5992	6529	6567	7053	8632			

P-3552

Figure 9 - Research and Development of the Vortex Valve Principle and its Application to a Hot-Gas (5500°F) Secondary Injection Thrust Vector Control System

APPENDIX A

CALCULATION OF TRANSIENT TEMPERATURES AT VARIOUS PARTS OF A SITVC VORTEX VALVE

1. INTRODUCTION

The object of this analysis is to find the time-temperature profile at various parts in a vortex valve after a 2000°F gas has been admitted through the control port.

In the SITVC system, the main flow is 5500°F aluminized hot gas and the control flow is 2000°F gas. As the 5500°F gas flows through the vortex power valve, it gives up heat to the valve and there is the possibility that the heat loss might lower the gas temperature enough to cause some of the aluminum to plate out and plug or restrict the valve passages. The maximum heat loss, of course, occurs during the early stages of a firing test while the valve parts are still cold. To prevent possible internal plugging, it is intended to minimize the heat loss by admitting the 2000°F control gas ahead of the 5500°F so that the valve is preheated before the 5500°F aluminized gas enters. Thus the 5500°F gas will lose less thermal energy to the walls of the vortex valve than if it contacted the cold walls directly.

Before determining the required preheating time, it is necessary to compute the rate various parts of the vortex valve will heat up in response to the 2000°F gas. Determining this transient temperature response is the purpose of this analysis.

The equations presented are derived by the method of finite-differences to facilitate the calculating of the transient temperatures inside a vortex valve. The flow system considered consists of 2000°F gas flowing through an insulated conduit, through an annulus, and then flowing radially between two discs. Using the special case of a uniform flow temperature, the transient temperatures at various parts in the valve are presented. The equations are general and applicable to any flow system involving heat transfer between the fluid and the walls.

The parts of concern are the inner liner, the housing structure and the center button of the valve, as shown by the designations 1 2

3 4 and 5 in Figure 1. The liners are backed by carbon-phenolic insulation and the button is hollow and symmetric in configuration. The equations assume that the tungsten parts are perfectly insulated and the control gas temperature at the wetted surface of the parts is 2000°F. The material of these parts is silver-infiltrated tungsten (15 percent by volume), and the secondary hot gas is to be generated by the Olin-Mathieson "OMAX" 453D, which is an ammonium nitrate composite propellant.

This report presents the results of analysis of the transient flow system. Dusinberre's finite-difference method (Ref. 1) has been followed. The results allow determination of the history (time variable) of the temperature distributions (spatial variable) in the flow and in the conduit walls as well.

The derived equations do not accommodate the phase-change of the infiltrant-silver, which has a melting point of 1760°F, but a method is discussed to compensate for the effect of latent heat, should the wall temperature exceed the melting point of the infiltrant material.

From the nature of the problem, the equations generally require a computerized program to obtain accurate results because of the involved repetitive calculations and the need to satisfy the stability criteria of the numerical method. As a special case, the steps are drastically simplified when the flow temperature is maintained constant throughout its passage. This situation may be realized when the flow velocity is relatively large and the length of the conduit wall of concern is relatively short.

2. ANALYSIS

Two general cases are to be studied, namely: (1) Two concentric cylinders with a hot gas flowing through the annular space, and (2) a hot gas flowing radially between a pair of concentric parallel discs.

The following assumptions and restrictions are imposed to the theoretical models:

(1) The flow has a constant temperature at the entrance, and a uniform temperature across its cross-section. The heat transfer in the direction parallel to the fluid flow is negligible.

(2) The wall thicknesses are thin, and there exists no temperature gradient in the walls in the direction

perpendicular to the fluid flow. Hence, the wall temperatures may be represented by the average local temperatures of the mass.

(3) The unexposed back faces of the walls are perfectly insulated, i.e., the insulator absorbs no heat and the material does not undergo endothermic ablative process. Neither does the wall material undergo phase-change within the range of operating temperatures.

2.1 A Concentric Double-Cylinder System with a Hot Gas Flowing Through the Annular Space (Figure 2)

The derivation may be proceeded following the finite-difference method, by an energy balance between the segmental wall element and the flow. The resulting equations are as follows:

For the inner cylinder ($r_1 \leqslant r \leqslant r_2$):

$$T_{ni}^{\Delta t} = \frac{K_i}{2} (T_m + T_{m+1}) \Delta t + (1 - K_i \Delta t) T_{ni} \tag{1}$$

where

$$K_i = \frac{2 h_i r_2}{C_{pi} \rho_i (r_2^2 - r_1^2)} \tag{2}$$

For the outer cylinder ($r_3 \leqslant r \leqslant r_4$):

$$T_{no}^{\Delta t} = \frac{K_o}{2} (T_m + T_{m+1}) \Delta t + (1 - K_o \Delta t) T_{no} \tag{3}$$

where

$$K_o = \frac{2 h_o r_3}{C_{po} \rho_o (r_4^2 - r_3^2)} \tag{4}$$

For the flow ($r_3 \leqslant r \leqslant r_2$):

A. Blatter 25 April 1966

$$\frac{1}{2}\left(T_{no}^{\Delta t} + T_{ni}^{\Delta t}\right) = \frac{2\pi\,\Delta t}{C_{pf}\rho_f A_f}\left(h_o r_3 T_{no} + h_i r_2 T_{ni}\right) + \frac{\Delta t}{\rho_f A_f}\left(\frac{\dot{m}}{\Delta x} - \frac{h_o r_3 + h_i r_2}{C_{pf}}\right) T_m$$

$$+ \left[1 - \frac{\Delta t}{\rho_f A_f}\left(\frac{\dot{m}}{\Delta x} + \frac{\pi h_o r_3 + \pi h_i r_2}{C_{pf}}\right)\right] T_{m+1} \qquad (5)$$

where

$$A_f = \pi\left(r_3^{\,2} - r_2^{\,2}\right) \qquad (6)$$

It is to be noted that the criteria of stability for calculation are:

$$\Delta t \leqslant \frac{{}_f A_f}{\dfrac{\dot{m}}{\Delta x} + \dfrac{\pi}{C_{pf}}\left(h_o r_3 + h_i r_2\right)} \qquad (7)$$

and

$$\Delta x \leqslant \frac{\dot{m}\, C_{pf}}{\left(h_o r_3 + h_i r_2\right)} \qquad (8)$$

The alternative expression which is an approximation of Eq. (5) is of the form:

$$T_{m+1} = \frac{2}{\left(r_3 h_o + r_2 h_i\right) + \dfrac{\dot{m} C_{pf}}{\pi\,\Delta x}}\left[r_3 h_o T_{no} + r_2 h_i T_{ni} + \left(\frac{\dot{m} C_{pf}}{2\pi\,\Delta x} - \frac{r_3 h_o + r_2 h_i}{2}\right) T_m\right] \qquad (9)$$

Two special cases may be deduced from the preceding general equations: first, for a single cylinder with a hot gas flowing externally; and second, for a single cylinder with a hot gas flowing internally. The following changes are necessary.

2.1.1 A Single Cylinder with a Hot Gas Flowing Externally*

For the cylinder, Eqs. (1) and (2) remain valid; but for the flow, Eq. (5) should be used with the following substitutions that:

$$\begin{cases} T_n^{\Delta t} = \frac{1}{2}\,(T_{no}^{\Delta t} + T_{ni}^{\Delta t}) \\ h_o = 0 \end{cases} \tag{10}$$

If Eq. (9) is used instead of Eq. (5), it requires to impose only that

$$h_o = 0 \tag{11}$$

2.1.2 A Single Cylinder with a Hot Gas Flowing Internally

For the cylinder, Eqs. (3) and (4) still apply; but for the flow, Eq. (5) should be used with the following substitutions that:

$$\begin{cases} T_n^{\Delta t} = \frac{1}{2}\,(T_{no}^{\Delta t} + T_{ni}^{\Delta t}) \\ h_i = 0 \end{cases} \tag{12}$$

If Eq. (9) is used instead of Eq. (5), it requires to impose only that:

$$h_i = 0 \tag{13}$$

2.2 A Hot Gas Flowing Radially Between a Pair of Concentric Parallel Discs (Figure 3)

For either disc, the temperature expression is:

*In fact, the flow is still in an annular space between two concentric cylinders, but only the inner cylinder participates in the heat transfer process effectively; and the outer cylinder is a perfect insulator which absorbs no heat under idealization.

$$T_n^{\Delta t} = \frac{K_w}{2} (T_m + T_{m+1}) \Delta t + (1 - K_w \Delta t) T_n \qquad (14)$$

where

$$K_w = \frac{h_w}{C_{pw} \rho_w b_w} \qquad (15)$$

For the flow, it has

$$\frac{1}{2} (T_{n1}^{\Delta t} + T_{n2}^{\Delta t}) = \frac{h_w \Delta t}{C_{pf} \rho_f} (t_{n1} + T_{n2}) + \frac{\Delta t}{\rho_f \ell} \left[\frac{\dot{m}}{\pi (r_{m+1}^2 - r_m^2)} - \frac{h_w}{C_{pf}} \right] T_m$$

$$+ \left[1 - \frac{\Delta t}{\rho_f \ell} \left\{ \frac{\dot{m}}{(r_{m+1}^2 - r_m^2)} + \frac{h_w}{C_{pf}} \right\} \right] T_{m+1} \qquad (16)$$

The alternative expression which is an approximation of Eq. (16) is of the form:

$$T_{m+1} = \frac{1}{1 + \dfrac{\dot{m} C_{pf}}{\pi (r_{m+1}^2 - r_m^2) h_w}} \left[(T_{n1} + T_{n2}) + \left\{ \frac{\dot{m} C_{pf}}{\pi (r_{m+1}^2 - r_m^2) h_w} - 1 \right\} T_m \right] \qquad (17)$$

2.3 Related Equations in Association with Calculations

Several quantities are to be determined before the numerical calculations are to be proceeded. They are included for reference.

2.3.1 Coefficients of Heat Transfer h, (Ref. 2)

For the turbulent flow:

$$h = 0.023 \left(\frac{\mu_f}{D_e G} \right)^{0.2} C_{pf} G \left(\frac{k}{C_p \mu} \right)_f^{2/3} \left(\frac{\mu_f}{\mu_w} \right)^{0.14} \qquad (18)$$

in which the subscript f refers to the bulk of the fluid, and w the wall of the flow conduit. To simplify the calculational procedure, it is desirable to separate the temperature-dependent from the independent factors. This can be achieved by substituting the state and continuity equations to Eq. (18) and resulting in

$$h = 0.023\, C_p \mu^{0.2}\, Pr^{-0.6}\, G^{0.8}\, D_e^{-0.2} \left(\frac{T_w}{T_f} \right)^{0.8} \tag{19}$$

For the laminar flow in an annular space:

$$h = \frac{k}{D_e} \left(\frac{D_o}{D_i} \right)^{0.8} \left(\frac{\dot{m} C_p}{k\, L} \right)_f^{0.45} \left(Gr \right)^{0.05} \tag{20}$$

2.3.2 The Equivalent Diameter D_e

By definition, the equivalent diameter is given by:

$$D_e = 4\, r_h = 4 \times \frac{\text{the cross-sectional area of the conduit}}{\text{the wetted perimeter}} \tag{21}$$

Thus, for an annular it is:

$$D_e = D_o - D_i \tag{22}$$

where $D_o = 2r_3$ and $D_i = 2r_2$ as shown in Figure 2.

The equivalent diameter for the fluid flowing radially between two concentric discs is found to be:

$$D_e = 2\delta \tag{23}$$

2.3.3 The Equivalent Thermal Conductivity, Density, and Specific Heat of an Infiltrated Metal (Porous Tungsten with Silver Infiltrant)

By Grootenhuis' expression (Ref 3):

$$k = f\, (k)_{silver} + (1-f)\, (k)_{tungsten} \tag{24}$$

and

$$C_p \rho = f\,(C_p \rho)_{silver} + (1-f)\,(C_p \rho)_{tungsten} \tag{25}$$

where

$$f = \frac{\text{the void volume filled with the infiltrant}}{\text{total volume of the metal matrix}} \tag{26}$$

3. NUMERICAL ILLUSTRATION AND RESULTS

The calculations will be carried out on the basis of the following input data, simplifications, and restrictions:

(1) The secondary hot gas of 2000 $^{\circ}$F entering at P (Fig. 1) with a constant flow rate $\dot{m}$ (lb/hr) splits equally, and enters the annular cylindrical space and the space between the two concentric discs.

(2) The hot gas is supplied continuously over the entire duration of operation. The temperature of the hot gas may be assumed constant throughout the conduits.

(3) The tapered portion 1 may be treated as a horizontal cylinder with the fluid flowing externally, but an equivalent radius should be used.

(4) The flow pattern between the two concentric discs is radial and uniform.

(5) Each region is isolated thermally, and there is no mutual conduction from each other.

The idealized models with dimensions are shown in Figure 4, and the physical properties of the wall material and the hot gas are given in Table 1. The equivalent diameters D_e, Reynolds numbers Re, the flux of the rate of flow G, and the average coefficient of heat transfer h are summarized in Table 2. It is to be noted that the h values were calculated with $T_w/T_f = 1$, which is a conservative estimation.

With $\Delta t = 1$ sec as the time increment and the overall length of each region as the segmental length, the numerical results of the average temperature response at various parts in the wall of the vortex valve were computed. They are presented in Table 3. The computational procedure is greatly simplified for the present application of a constant flow temperature; otherwise, it requires to solve the unknown temperatures of the wall and the flow simultaneously. Then, a computer calculation is necessary. In that connection, finer values of the time increment Δt and the length increment Δx may be used if higher accuracy is desired.

4. ESTIMATION OF THE EFFECT OF LATENT HEAT OF THE INFILTRANT-SILVER

The preceding analysis has not taken the effect of the phase-change of the infiltrant into account. For silver, its melting point is 1760°F and its latent heat of fusion is L_f=26.5 (cal/g) = 47.71(Btu/lb). Therefore, certain parts, 5 for example, has exceeded the melting point and other parts would possibly reach that temperature, if a longer duration of operation should be considered. A method of calculation is outlined for a quick estimation of the effect of neglecting the latent heat to the actual temperature response of the infiltrated metal.

Let T_{mp} be the melting point of the infiltrant, T' is the temperature of the infiltrant metal that would respond upon absorbing the heat energy by neglecting the phase-change of the impregnated infiltrant, the energy stored in the infiltrated metal is of the amount

$$\frac{(C_p \rho V)_{\text{overall metal}} (T' - T_{mp})}{\Delta t}$$

and this amount of energy, in fact, is consumed by the phase-change of the infiltrant which has the latent heat of fusion L_f, then

$$\frac{L_f (\rho V)_{\text{infiltrant}}}{\Delta t}$$

Equating the preceding expressions gives

$$T = T' - T_{mp} = \frac{L_f (\rho V)_{\text{infiltrant}}}{(C_p \rho V)_{\text{overall metal}}} \qquad (27)$$

but from Eqs. (25) and (26),

$$\frac{V_{infiltrant}}{V_{overall\ metal}} = f$$

and

$$(C_p \rho)_{overall\ metal} = f\,(C_p \rho)_{infiltrant} + (1-f)\,(C_p \rho)_{matrix\ metal}$$

then Eq. (27) may be rewritten as

$$T = T' - T_{mp} = \frac{L_f}{(C_p)_{infiltrant} + \left(\frac{1-f}{f}\right)\frac{(C_p \rho)_{matrix\ metal}}{\rho_{infiltrant}}} \tag{28}$$

For f = 15%, infiltrant = silver and matrix metal = tungsten, using data as listed in Table 1, one has

$$T = T' - 1760 = \frac{47.7}{0.0744 + \frac{0.85}{0.15}\,\frac{0.042 \times 1204}{654}} = 93\ (^{o}F) \tag{29}$$

The result indicates that the temperatures as calculated on the basis of the equations given in Section 2 are in error by an amount of 93°F as soon as the overall temperature of the metal reaches the melting point of the infiltrant. The accuracy of such estimation must be confined to a narrow band centering around the melting point of the infiltrant, and a reasonable short period of operational duration. Because, in reality, the molten infiltrant will not remain in the porous voids for any length of time.

Hwa-Ping Lee

Hwa-Ping Lee

HPL:mp

Attachment: Nomenclature; References; Figures 1, 2, 3 and 4; and Tables 1, 2, and 3.

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NOMENCLATURE

A_f	Cross-sectional area of fluid in a conduit, (ft^2)
b_w	Wall thickness of disc, (ft)
C_p	Specific heat at constant pressure, (Btu/lb-^oF); C_{pf}, C_{pi}, C_{po}, and C_{pw} refer to that of fluid, inner wall, outer wall, and disc, respectively.
D_e	Equivalent diameter equals $4r_h$, (ft)
f	Ratio of void volume filled with infiltrant to total volume of matrix of metal
G	Mass velocity of fluid, $(lb/hr\text{-}ft^2)$
h	Coefficient of convective heat transfer between fluid and surface, (Btu/hr-ft^2-^oF); h_i, h_o, and h_w refer to that between fluid and inner wall, outer wall, and disc respectively.
K	Group constant, (1/hr); as defined in Eq.(2) for K_i, Eq.(4) for K_o, and Eq.(15) for K_w.
k	Thermal conductivity, (Btu/hr-ft-^oF).
L_f	Latent heat of infiltrant, (Btu/lb)
ℓ	Spacing between two parallel discs, (ft)
$\dot{m}$	Mass Rate of flow, (lb/hr)
r	Radius, (ft)
V	Volume, (ft^3)

NOMENCLATURE (Cont.)

r_h	Hydraulic radius equals cross-section divided by total wetted perimeter, (ft)
T_n	Average temperature in the cylinder wall of the nth segment, (^oF)
T_m	Temperature in the flow at the mth station, (^oF)
$T^{\Delta t}$	Temperature response after a time increment Δt, (^oF)
Δt	Time increment, (hr)
Δx	Length of segment of cylinder, (ft)
ρ	Density, (lb/ft^3); ρ_f, ρ_i, ρ_o, and ρ_w refer to that of fluid, inner wall material, outer wall material, and disc material, respectively.
μ	Viscosity, (lb/hr-ft)
Re	Reynolds number equals to D_eG/μ
Pr	Prandtl number equals to $C_p\mu/k$
Gr	Grashof number equals to $(D^3\rho^2 g/\mu^2)(\beta\Delta t)$

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(2) McAdams, W. H., "Heat Transmission", McGraw-Hill Book Co., 3rd Edition, 1954.

(3) Groothenhuis, P., "The Mechanism and Application of Diffusion Cooling", J. Royal, Aeronautics Society, Vol. 63, pp. 73-89, 1959.

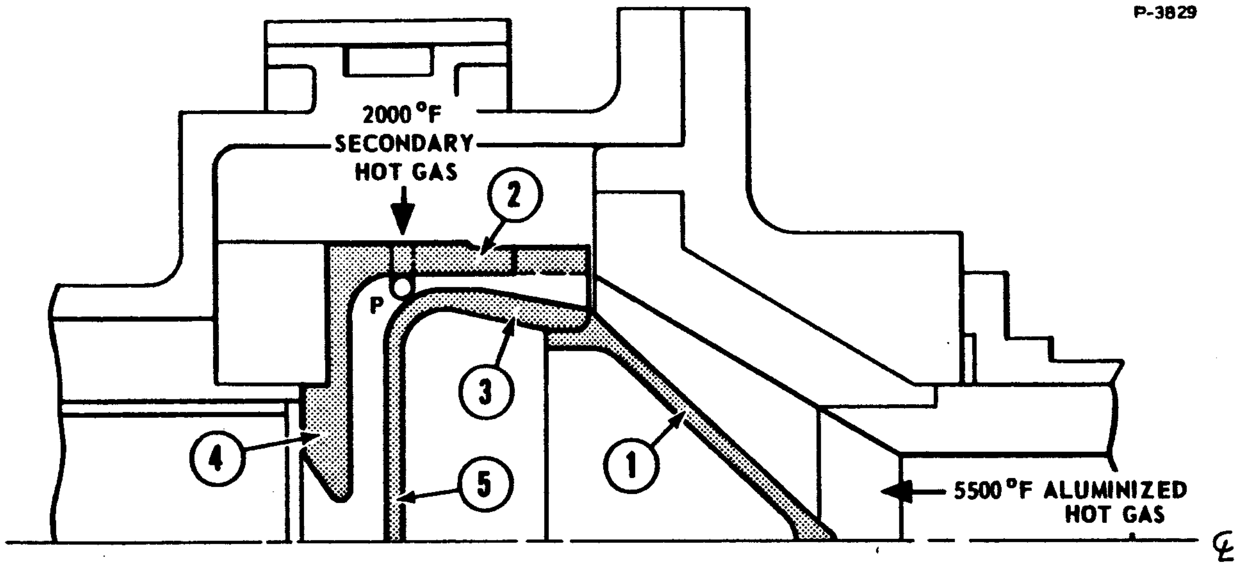

Figure 1 - Physical Model of the SITVC Vortex Valve

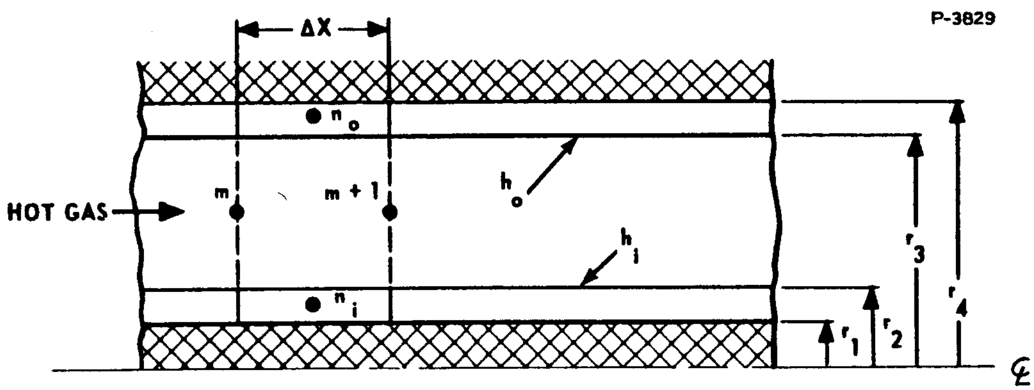

Figure 2 - Model and Nomenclature of the Concentric Cylinders with the Hot Gas Flowing in the Annular Space

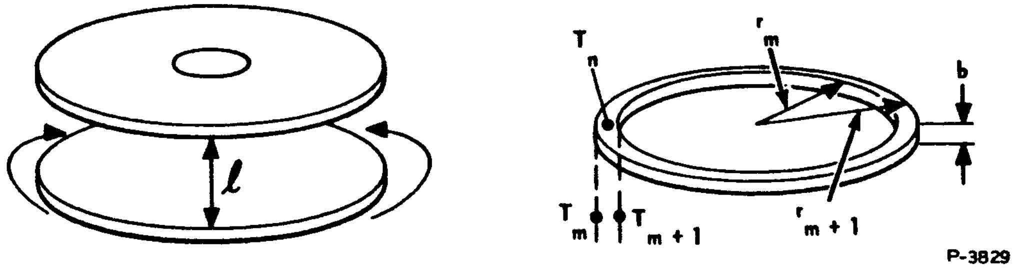

Figure 3 - Model and Nomenclature of the Concentric Parallel Discs

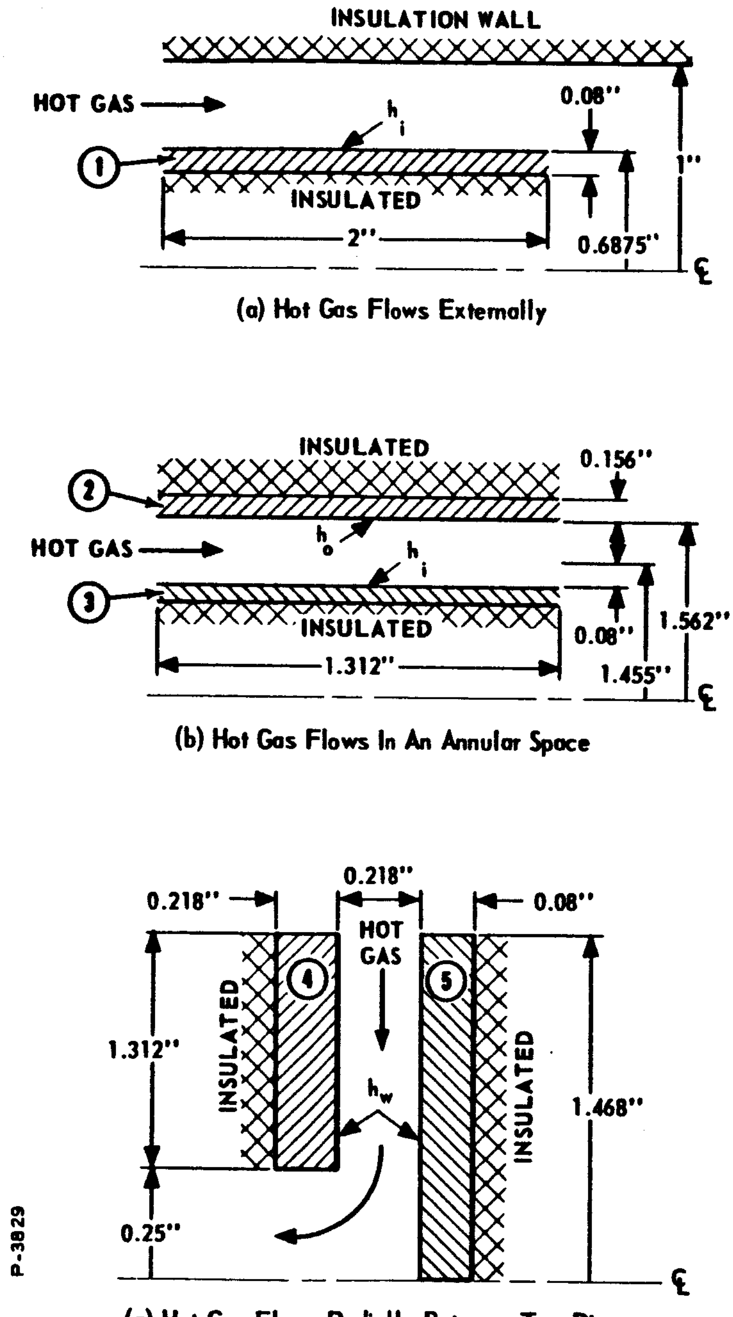

Figure 4 - Idealized Models and Dimensions

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TABLE 1 PHYSICAL PROPERTIES OF THE WALL MATERIAL AND THE HOT GAS

Medium	Pure Tungsten	Silver	Silver Infiltrated Tungsten (f=15%)	Hot Gas
Density, ρ (lb/ft^3)	1204	654	$(C_p\rho)=48.3$ $(Btu/ft^3\text{-}^oF)$ Eq. (25)	0.146
Specific Heat, Cp $(Btu/lb\text{-}^oF)$	0.04	0.0744		0.473
Thermal Conductivity k $(Btu/hr\text{-}ft\text{-}^oF)$	70	200	89.5 Eq. (24)	0.14
Viscosity, (lb/ft-hr)	-	-	-	0.2
Melting Point, (^oF)	6170	1760	-	-

TABLE 2 CALCULATED RESULTS OF VARIABLES

$\dot{m}$ = 0.21 lb/sec = 755 lb/hr p = 200 psi

Region	1	2 - 3	4 - 5
D_e, (ft)	0.104	0.0357	0.03635
G, $(lb/hr\text{-}ft^2)$	32,850	53,850	185,500
Re	17,100	9,605	33,700
h, $(Btu/ft^2\text{-}hr\text{-}^oF)$	64.4	116.7	318

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TABLE 3 SUMMARY OF THE AVERAGE TRANSIENT TEMPERATURES (^{o}F) IN VARIOUS PARTS IN THE VORTEX VALVE WITH A UNIFORM FLOW TEMPERATURE OF 2000^{o}F

Time t (sec)	1	2	3	4	5
0	70	70	70	70	70
1	298	165	261	263	600
2	402	257	433	437	985
3	500	344	588	593	1264
4	591	432	728	733	1467
5	677	510	853	860	1614
6	757	587	966	974	1721
7	833	660	1068	1067	1799
8	903	730	1159	1160	1856
9	969	797	1241	1243	1897
10	1032	861	1316	1320	1926

APPENDIX B

STABILITY ANALYSIS OF A PRESSURE REGULATOR FOR DUAL-SPGG SYSTEMS

The purpose of this analysis is to determine the stability criteria of a hot gas pressure regulator for controlling the pressure level of one gas generator as a function of the pressure level in a second gas generator. The system schematic is shown in Figure 1, and the basic system constants are summarized in Table 1. A parametric analysis will be made first, in order to determine basic sizing; then the system will be dynamically analyzed.

PARAMETRIC ANALYSIS

$$P_1 A_1 + P_4 (A_2 - A_1) = P_2 A_2 \tag{1}$$

$$A_2 = A_1 \left(\frac{P_1 - P_4}{P_2 - P_4} \right) \tag{2}$$

where

$A_1 = 0.0311 \text{ in}^2$

$P_1 = 2215 \text{ psia}$

$P_2 = 395 \text{ psia}$

$P_4 = 15 \text{ psia}$

$$A_2 = 0.0311 \left(\frac{2215 - 15}{395 - 15} \right)$$

$$A_2 = 0.180 \text{ in}^2$$

$$D_2 = 0.479 \text{ in}$$

Table 1 - Summary of Hot Gas Pressure Regulator Data

SPGG-1	Pressure Regulator		SPGG-2
	Side 1	Side 2	
C_{21} = 0.412 °R/sec	D_1 = 0.199 in	D_2 = 0.479 in	C_{22} = 0.532 °R/sec
k_1 = 1.28	X_1 = 0.012 in	A_2 = 0.180 in^2	k_2 = 1.132
R_1 = 1009 in/°R	A_1 = 0.0311 in^2		R_2 = 702 in/°R
A_g = 79 in^2	(Pressure Area)		
K_g = 0.0249 in^3/#sec			
r = 0.127 in/sec			
P = 2365 psia			
n = 0.21			
γ_g = 0.053 #/in^3			

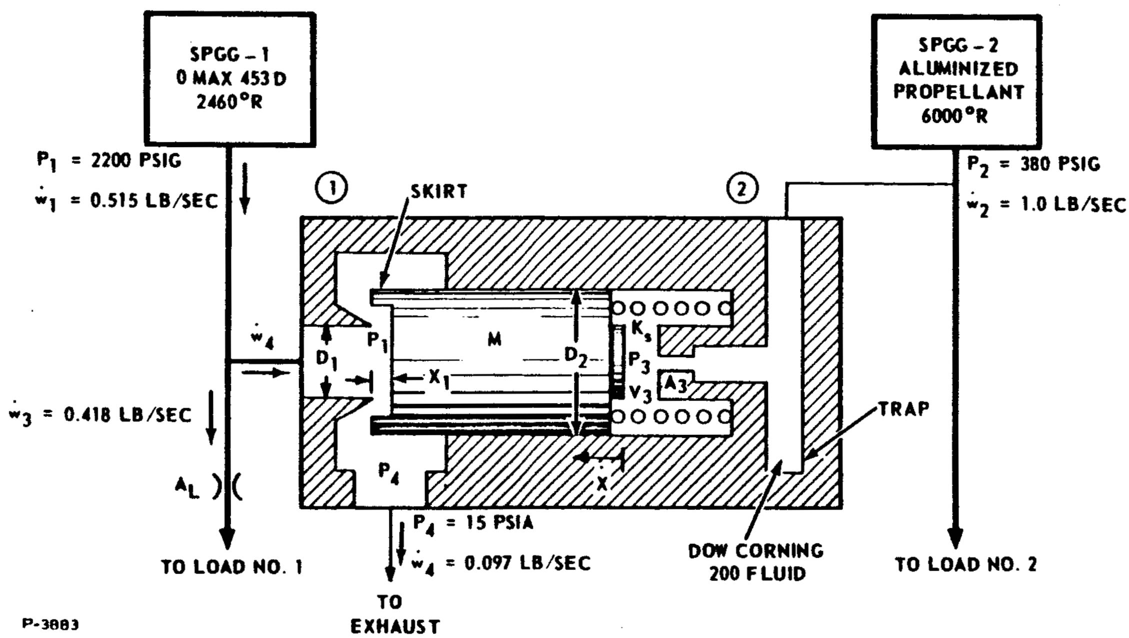


Figure 1 - Hot Gas Pressure Regulator Schematic

Compute Nominal Valve Position

$$\dot{w}_4 = \frac{C_d C_2}{\sqrt{T}} \pi D_1 X_1 P_1 f_1 \left(\frac{P_4}{P_1} \right) \tag{3}$$

Assuming sonic flow,

$$X_1 = \frac{\dot{w}_4 \sqrt{T}}{C_d C_2 \pi D_1 P_1} \tag{4}$$

where

$\dot{w}_4$ = 0.097 lb/sec

T = 2460°R, $\sqrt{T}$ = 49.6 $\sqrt{°R}$

C_d = 0.7

C_2 = 0.412 $\sqrt{°R}$/sec

D_1 = 0.199 in

P_1 = 2215 psia

$$X_1 = \frac{0.097 \times 49.6}{0.7 \times 0.412 \times \pi \times 0.199 \times 2215}$$

$$X_1 = 0.01208 \text{ in}$$

DYNAMIC ANALYSIS

This is a small-perturbation analysis. Equations are developed for small displacements of the poppet and small variations of pressure and flow.

Force Balance on Poppet (Neglecting Friction)

Assume P_4 = 0 psia for the dynamic analysis. The force balance is

$$K_s (X-X_1) + P_3 A_2 - P_1 A_1 = M\ddot{X} \tag{5}$$

where $X-X_1$ is displacement of poppet resulting from a change in P_2. For this analysis, P_2 is assumed to rise, so that X is less than X_1 and the term $K_s (X-X_1)$ is negative.

Flow of Dow Corning Fluid Through Orifice A3

Neglect compressibility of Dow Corning Fluid in volume V_3.

$$C_{d3} A_3 \sqrt{\frac{2g}{\gamma_d} (P_2 - P_3)} = A_2 \dot{X} \tag{6}$$

Conservation of Flow Out of SPGG-1

$$\dot{w}_1 = \dot{w}_3 + \dot{w}_4 \tag{7}$$

SPGG-1 Flow Is Pressure Dependent

$$\dot{w}_1 = A_g K_g P_1^{\,n} \gamma_g \tag{8}$$

Load Flow, Sonic

$$\dot{w}_3 = \frac{C_{dL} C_{21}}{\sqrt{T_1}} A_L P_1 \tag{9}$$

Hot Gas Regulator Flow, Sonic

$$\dot{w}_4 = \frac{C_{d4} C_{21}}{\sqrt{T_1}} \pi D_1 X P_1 \tag{10}$$

Substitute (8), (9) and (10) into (7)

$$A_g K_g P_1^{\,n} \gamma_g = \frac{C_{dL} C_{21} A_L P_1}{\sqrt{T_1}} + \frac{C_{d4} C_{21} \pi D_1 X P_1}{\sqrt{T_1}} \tag{11}$$

Differentiate Equation (11)

$$A_g K_g \gamma_g n P_{1o}^{(n-1)} \overline{P}_1 = \frac{C_{dL} C_{21} A_L}{\sqrt{T_1}} \overline{P}_1 + \frac{C_{d4} C_{21} \pi D_1}{\sqrt{T_1}} X_o \overline{P}_1$$

$$+ \frac{C_{d4} C_{21} \pi D_1}{\sqrt{T_1}} P_{1o} \overline{X} \tag{12}$$

With the regulator reacting to a change in P_2, $\overline{X}$ is negative when $\overline{P}_1$ is positive (and vice versa). Therefore, equation (12) becomes

$$A_g K_g \gamma_g n P_{1o}^{(n-1)} \overline{P}_1 = \frac{C_{dL} C_{21} A_L}{\sqrt{T_1}} \overline{P}_1 + \frac{C_{d4} C_{21} \pi D_1}{\sqrt{T_1}} X_o \overline{P}_1$$

$$- \frac{C_{d4} C_{21} \pi D_1}{\sqrt{T_1}} P_{1o} \overline{X} \qquad (12')$$

Solve for $\overline{X}$

$$\overline{X} = \frac{1}{P_{1o}} \left(- \frac{A_g K_g \gamma_g n P_{1o}^{(n-1)} \sqrt{T_1}}{C_{d4} C_{21} \pi D_1} + \frac{A_L}{\pi D_1} + X_o \right) \overline{P}_1 \qquad (13)$$

Operate on Equations (5) and (6) to Eliminate P_3

Differentiate (6)

$$C_{d3} A_3 \sqrt{\frac{2g}{\gamma_d}} \frac{1}{2} (P_{2o} - P_{3o})^{-\frac{1}{2}} (\overline{P}_2 - \overline{P}_3) = A_2 \dot{\overline{X}} \qquad (14)$$

Apply Laplace Operator

$$\frac{C_{d3} A_3}{2 \sqrt{P_{2o} - P_{3o}}} \sqrt{\frac{2g}{\gamma_d}} (\overline{P}_2 - \overline{P}_3) = A_2 \overline{X} S \qquad (15)$$

where

$S = d/dt \; (sec^{-1})$

Solve for $\overline{P}_3$

$$\overline{P}_3 = \overline{P}_2 - \frac{A_2}{A_3 C_{d3}} \sqrt{\frac{2 \gamma_d (P_{2o} - P_{3o})}{g}} S \overline{X} \qquad (16)$$

Rewrite Equation (5) and Differentiate

$$P_3 A_2 - P_1 A_1 = M\ddot{X} - K_s X + K_s X_1 \tag{5}$$

$$A_2 \bar{P}_3 - A_1 \bar{P}_1 = (MS^2 - K_s)\bar{X} \tag{17}$$

Substitute (16) into (17)

$$A_2 \left[\bar{P}_2 - \frac{A_2}{A_3 C_{d3}} \sqrt{\frac{2\gamma_d}{g} (P_{2o} - P_{3o})}\, S \bar{X} \right] - A_1 \bar{P}_1 = (MS^2 \cdot K_s)\bar{X} \tag{18}$$

<u>Solve Equation (18) for $\bar{X}$</u>

$$\bar{X} = \frac{A_2 \bar{P}_2}{MS^2 + \frac{A_2^2}{A_3 C_{d3}} \sqrt{\frac{2\gamma_d}{g}(P_{2o} - P_{3o})}\, S - K_s} - \frac{A_1 \bar{P}_1}{MS^2 + \frac{A_2^2}{A_3 C_{d3}} \sqrt{\frac{2\gamma_d}{g}(P_{2o} - P_{3o})}\, S - K_s} \tag{19}$$

<u>Combine (13) and (19), Eliminating $\bar{X}$</u>

$$\frac{1}{P_{1o}} \left(- \frac{A_g K_g \gamma_g n P_{1o}^{(n-1)} \sqrt{T_1}}{C_{d4} C_{21} \pi D_1} + \frac{A_L}{\pi D_1} + X_o \right) \bar{P}_1$$

$$= \frac{A_2 \bar{P}_2}{MS^2 + \frac{A_2^2}{A_3 C_{d3}} \sqrt{\frac{2\gamma_d}{g}(P_{2o} - P_{3o})}\, S - K_s} - \frac{A_1 \bar{P}_1}{MS^2 + \frac{A_2^2}{A_3 C_{d3}} \sqrt{\frac{2\gamma_d}{g}(P_{2o} - P_{3o})}\, S - K_s} \tag{20}$$

Convert to $\overline{P}_1/\overline{P}_2$ Transfer Function

$$\frac{\overline{P}_1}{\overline{P}_2} = \frac{\dfrac{A_2}{MS^2 + \dfrac{A_2^{\,2}}{A_3 C_{d3}}\sqrt{\dfrac{2\gamma_d}{g}(P_{2o} - P_{3o})}\, S - K_s}}{\dfrac{1}{P_{1o}}\left[- \dfrac{A_g K_g \gamma_g n P_{1o}^{(n-1)}\sqrt{T_1}}{C_{d4} C_{21} \pi D_1} + \dfrac{A_L}{\pi D_1} + X_o\right] + \dfrac{A_1}{MS^2 + \dfrac{A_2^{\,2}}{A_3 C_{d3}}\sqrt{\dfrac{2\gamma_d}{g}(P_{2o} - P_{3o})}\, S - K_s}} \tag{21}$$

Put in standard form

$$\frac{\overline{P}_1}{\overline{P}_2} = \frac{\dfrac{A_2 P_{1o}}{A_1 P_{1o} - K_s a}}{\dfrac{MaS^2}{P_{1o} A_1 - K_s a} + \dfrac{A_2^{\,2} a}{A_3 C_{d3} (P_{1o} A_1 - K_s a)}\sqrt{\dfrac{2\gamma_d}{g}(P_{2o} - P_{3o})}\, S + 1} \tag{22}$$

where

$$a = - \frac{A_g K_g \gamma_g n P_{1o}^{(n-1)} \sqrt{T_1}}{C_{d4} C_{21} \pi D_1} + \frac{A_L}{\pi D_1} + X_o \text{ (in.)}$$

Summary of Constants

A_1	$= 0.0311$ in^2	Pressure Area of Poppet
A_2	$= 0.180$ in^2	Pressure Area for SPGG-2
A_3	$= 0.00314$ in^2	Damping Orifice
A_g	$= 79$ in^2	Burning Area of Grain
A_L	$= 0.0325$ in^2	SPGG-1 Load Area

C_{d3}	=	0.7	Assumed Coefficient of Discharge for Orifice A_3
C_{d4}	=	0.7	Assumed Coefficient of Discharge
C_{21}	=	$0.412 \sqrt{°R}/sec$	Thermodynamic Gas Constant for OMAX 453D
D_1	=	0.199 in	Poppet Seat Diameter
g	=	$386\ in/sec^2$	Acceleration of Gravity
K_g	=	$0.0249\ in^3/lb\text{-}sec$	Grain Constant of OMAX 453D
K_s	=	100 lb/in	Spring Rate, Selected Minor Effect
M	=	$0.0000824\ lb\text{-}sec^2/in$	Poppet Mass
n	=	0.21	Pressure Exponent of OMAX 453D
P_{1o}	=	2215 psia	Nominal Pressure in SPGG-1
$P_{2o}-P_{3o}$	=	7 psia	Nominal Pressure Drop Across Orifice A_3
T_1	=	2460°R	Gas Temperature
X_o	=	0.012 in	Nominal Poppet Valve Opening
γ_d	=	$0.036\ lb/in^3$	Density of Dow Corning Fluid
γ_g	=	$0.053\ lb/in^3$	Density of OMAX 453D Grain

<u>Determine A_L</u>

$$A_L = \frac{\dot{w}_3 \sqrt{T_1}}{C_{d1}\, C_{21}\, P_{1o}} \tag{23}$$

where

$\dot{w}_3 = 0.418$ lb/sec

$T_1 = 2460°R, \quad \sqrt{T_1} = 49.6 \sqrt{°R}$

$C_{d1} = 0.7$

$C_{21} = 0.412 \sqrt{°R}/sec$

$P_{1o} = 2215$ psia

$$A_L = \frac{0.418 \times 49.6}{0.7 \times 0.412 \times 2215}$$

$$A_L = 0.0325 \text{ in}^2$$

Determine a

$$a = -\frac{A_g K_g \gamma_g n P_{1o}^{(n-1)} \sqrt{T_1}}{C_{d4} C_{21} \pi D_1} + \frac{A_L}{\pi D_1} + X_o \tag{24}$$

$$= -\frac{79 \times 0.0249 \times 0.053 \times 0.21 \times (2215)^{-0.79} \times 49.6}{0.7 \times 0.412 \times \pi \times 0.199} + \frac{0.0325}{\pi \times 0.199} + 0.012$$

$$a = -0.0139 + 0.0520 + 0.012$$

$$a = 0.0501 \text{ in.}$$

Determine M

Poppet made from Alumina and Moly.

Alumina Mass:

$$M_a = \frac{W_a}{g} = \frac{V_a \gamma_a}{g} = \frac{0.7854\, D_2^2\, L_2\, \gamma_a}{g} \tag{25}$$

Specific Gravity of Alumina is 2.95

$$\gamma_a = \frac{2.95 \times 64.4}{1728} = 0.11 \text{ lb/in}^3 \tag{26}$$

$$D_2 = 0.479 \text{ in}$$

$$L_2 = 3\, D_2 = 3 \times 0.479 = 1.437 \text{ in}$$

$$g = 386 \text{ in/sec}^2$$

$$M_a = \frac{0.7854 \times (0.479)^2 \times 1.437 \times 0.11}{386}$$

$$= 0.0000735 \text{ lb-sec}^2\text{/in}$$

Moly Mass:

$$M_m = \frac{0.7854}{g} D_1^{\,2} L_1 \gamma_m \tag{27}$$

γ_m = 0.37 lb/in^2

D_1 = 0.199 in

L_1 = 1.5 D_1 = 1.5 x 0.199 = 0.298 in

$$M_m = \frac{0.7854 \times (0.199)^2 \times 0.298 \times 0.37}{386}$$

M_m = 0.00000888 lb-sec^2/in

$$M = M_a + M_m \tag{28}$$

$$M = 0.0000824 \text{ lb-sec}^2\text{/in}$$

<u>Compute γ_d</u>

$$\gamma_d = \text{Sp.Gr.} \times \frac{64.4}{1728} \tag{29}$$

Sp.Gr. = 0.968 Dow Corning 200 Fluid
μ_d = 100 cs (Viscosity)

$$\gamma_d = \frac{0.968 \times 64.4}{1728}$$

γ_d = 0.036 lb/in^3

<u>Compute Gain</u>

$$\text{Gain} = \frac{A_2 P_{1o}}{A_1 P_{1o} - K_s a} \tag{30}$$

$$= \frac{0.180 \times 2215}{0.0311 \times 2215 - 100 \times 0.0501}$$

$$\text{Gain} = \frac{398}{63.9} = 6.23$$

(Would be 5.78 psi/psi with no spring)

Compute Inertia Term

$$\frac{MaS^2}{P_{1o}A_1 - K_s a} = \frac{8.24 \times 10^{-5} \times 0.0501}{63.9} S^2$$

$$= 6.46 \times 10^{-8} S^2$$

Compute Natural Frequency

$$\omega_n = \sqrt{\frac{1}{6.46 \times 10^{-8}}} = \sqrt{0.155 \times 10^8} = 0.394 \times 10^4$$

ω_n = 3940 rad/sec = 627 cps Undamped Natural Frequency

Determine ζ

$$\frac{2\zeta}{\omega_n} = \frac{A_2^{\,2}\, a}{A_3 C_{d3} (P_{1o} A_1 - K_s a)} \sqrt{\frac{2\gamma_d}{g}(P_{2o} - P_{3o})} \qquad (31)$$

where

ω_n = 3940 rad/sec

A_2 = 0.180 in^2 $A_2^{\,2}$ = 0.0324 in^4

A_3 = 0.0031 in^2 D_3 = 0.063 in

a = 0.0501 in

$P_{1o} A_1 - K_s a$ = 63.9 lb

γ_d = 0.036 lb/in^3

g = 386 in/sec^2

C_{d3} = 0.7

Evaluate $P_{2o} - P_{3o}$

This term is evaluated by assuming the poppet will oscillate at the undamped natural frequency and at an amplitude equal to 10 percent of its nominal opening. From (6)

$$C_{d3} A_3 \sqrt{\frac{2g}{\gamma_d}(P_2 - P_3)} = A_2 \dot{X} \qquad (6)$$

$$C_{d3} A_3 \sqrt{\frac{2g}{\gamma_d}(P_2 - P_3)} = 0.1\, A_2\, X_1\, \omega_n \tag{32}$$

$$P_2 - P_3 = \left(\frac{0.1\, A_2\, X_1\, \omega_n}{C_{d3}\, A_3}\right)^2 \frac{\gamma_d}{2g} \tag{33}$$

where

X_1 = 0.012 in.

ω_n = 3940 rad/sec

$$P_2 - P_3 = \left(\frac{0.1 \times 0.180 \times 0.012 \times 3940}{0.7 \times 0.00314}\right)^2 \frac{0.036}{2 \times 386}$$

$$= (387)^2 \times 4.67 \times 10^{-5}$$

$$= 7 \text{ psi}$$

Or, more specifically,

$$P_{2o} - P_{3o} = 7 \text{ psi}$$

$$\zeta = \frac{\omega_n A_2^2 a}{2A_3\, C_{d3}\, (P_{1o} A_1 - K_s a)} \sqrt{\frac{2\, \gamma_d}{g}(P_{2o} - P_{3o})} \tag{34}$$

$$= \frac{3940 \times 0.0324 \times 0.0501}{2 \times 0.00314 \times 0.7 \times 63.9} \sqrt{\frac{2 \times 0.036}{386} \times 7}$$

$$= 22.8 \times \sqrt{13.06 \times 10^{-4}} = 22.8 \times 3.62 \times 10^{-2}$$

$$= 0.824$$

<u>Compute Damping Term</u>

$$\frac{2\, \zeta\, S}{\omega_n} = \frac{2 \times 0.824\, S}{3940} = 4.18 \times 10^{-4}\, S$$

Write Transfer Function

$$\frac{P_1}{P_2} = \frac{6.23}{6.46 \times 10^{-8} S^2 + 4.18 \times 10^{-4} S + 1} \quad \text{(With Spring)} \qquad (35)$$

$$\frac{P_1}{P_2} = \frac{5.78}{5.99 \times 10^{-8} S^2 + 3.83 \times 10^{-4} S + 1} \quad \text{(Without Spring)} \qquad (36)$$

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

Undamped Natural Frequency

ω_n = 627 cps with spring
652 cps without spring

Damping Factor

ζ = 0.824 with spring
0.784 without spring

The linearized system is a quadratic and is sufficiently fast and adequately damped for the intended application. Friction effects were not analyzed, however, and therefore additional damping might be desirable to ensure stability. Damping can be increased by:

1. Reducing the size of orifice A_3.
2. Using a fluid of higher viscosity.

These two parameters have relatively small effect on other performance considerations. The system is actually slightly oscillatory because of the nonlinear character of the damping term. A computer study would be required for an exact solution yielding poppet amplitude and frequency. It has been established, however, that the damping is sufficient to limit the amplitude of oscillations to less than 10 percent of its nominal poppet position. The mechanical spring, K_s, is not essential to the operation of the regulator and can be eliminated if desired.

APPENDIX C

TEST PLAN FOR 5500°F SITVC SINGLE VALVE SYSTEM

The system to be tested is a single vortex valve 5500°F SITVC system with a 2000°F control stage. The major objective of the test is to modulate the 5500°F gas with a vortex valve and 2000°F gas. The secondary test objective is to verify the integrity of the valve design.

The test is to be conducted in two parts. The first part is a steady-state cold gas test of the system with N_2 used as supply and control gases. The second part of the test is a steady-state hot gas test of the system with the 5500°F SPGG used as the hot gas supply source and the 2000°F SPGG used as the hot gas control source.

Cold Gas Test

The purpose of the cold gas test is to determine the steady-state performance characteristics of the single vortex valve SITVC system and to verify the intended performance of the system's various components (control valve, vortex valve, sequencer, etc.) prior to the hot gas test. The cold gas test will be conducted in three parts as described below.

Control Valve Performance

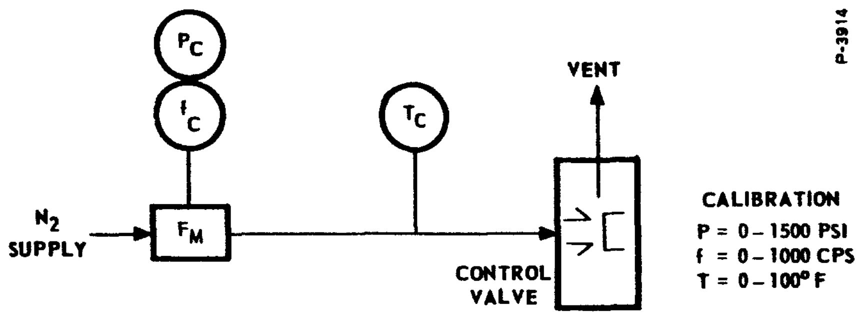
The control valve performance will be determined from data recorded from a test setup as shown in Figure 1(a). The test will be run with P_c varying from 0 to 1500 psig. The recorded data will be used to obtain valve flow rate, $\dot{w}$, vs control pressure, P_c.

Vortex Valve Performance

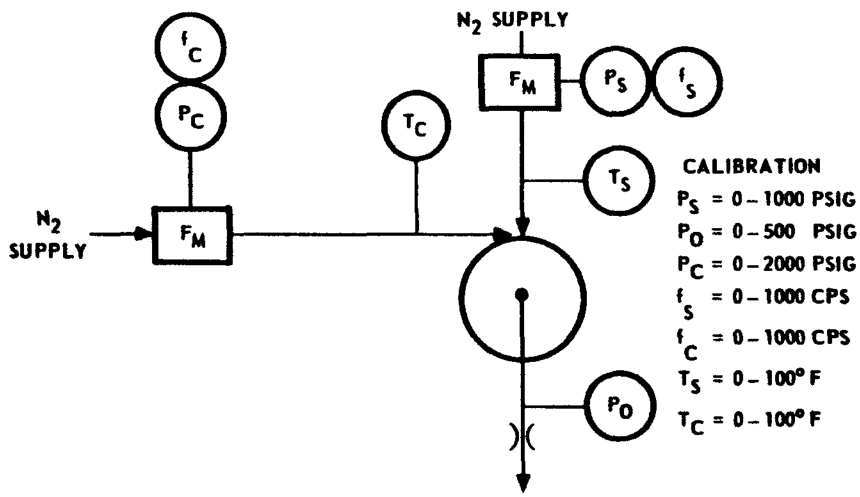
The vortex valve performance will be determined from data recorded from a test setup as shown in Figure 1(b). The test will be run three times with P_s regulated at 950, 500, and 700 psig and P_c modulated through ranges required for full valve turndown. The recorded data will be used to obtain the following performance characteristics:

$$\dot{w}_o \text{ vs } P_o$$

$$\frac{w_o}{w_{o_{max}}} \text{ vs } P_c$$



(a) Test Schematic For Cold Gas Test Of Control Valve



(b) Test Schematic For Cold Gas Test Of Vortex Valve

Figure 1 - Component Performance Test Schematics

Single Vortex Valve SITVC System Performance

The single vortex valve SITVC system performance characteristics will be determined from data recorded from a test setup as shown in Figure 2.

The cold gas test will be set up and run in a manner such to approximate the hot gas test of the same system. The cold gas supplied to the SPGG vent and vortex valve combination will be at a constant flow rate as controlled by the orifice upstream of the empty SPGG. The value of P_s, when the vortex valve is fully turned down, will be 950 psig. The vortex valve control pressure, P_c, will be modulated by the control valve whichin turn received an input signal from the solenoid valve. The solenoid valve will be actuated by the sequencer that is to be used in the hot gas test.

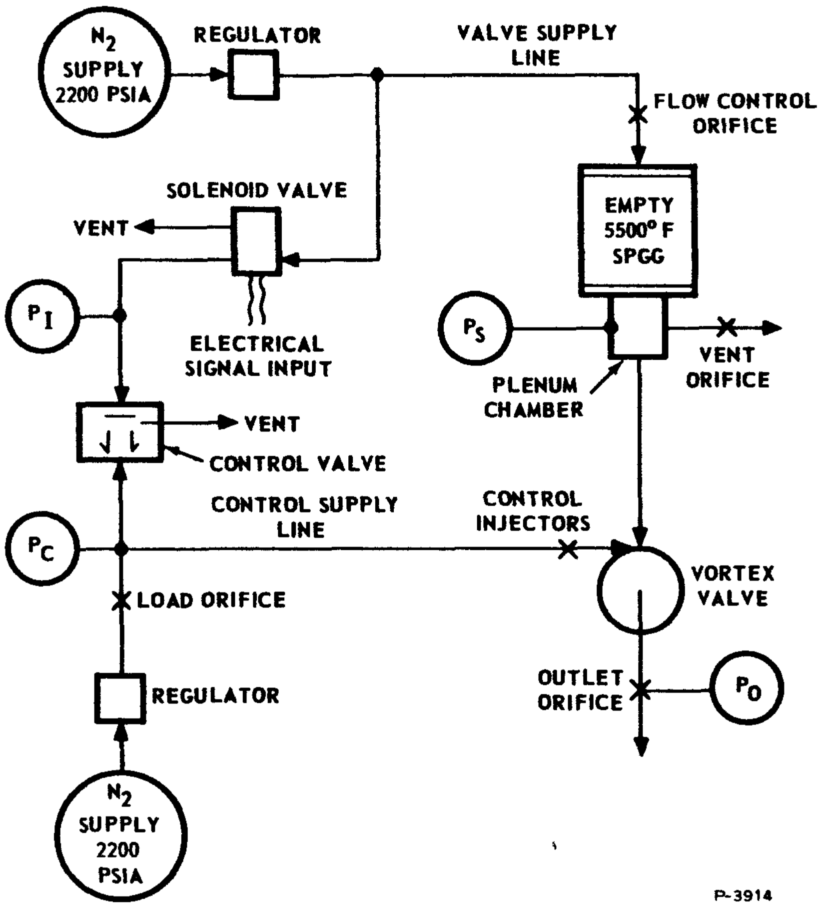


Figure 2 - Test Schematic for Cold Gas Test of 5500°F SITVC System - Single Vortex Valve

The data recorded in this portion of the cold test will indicate the compatibility of the system's many components and suggest any necessary system changes required prior to the hot test.

Hot Gas Test

The test schematic for the hot gas test is shown in Figure 3. This test will provide information on the ability of the vortex valve to modulate 5500°F gas.

The 5500°F SPGG will be used to supply the vortex valve and the 2000°F SPGG will provide the control gas. The ignition of the SPGGs, the modulation of the control gas by the solenoid valve, and the starting

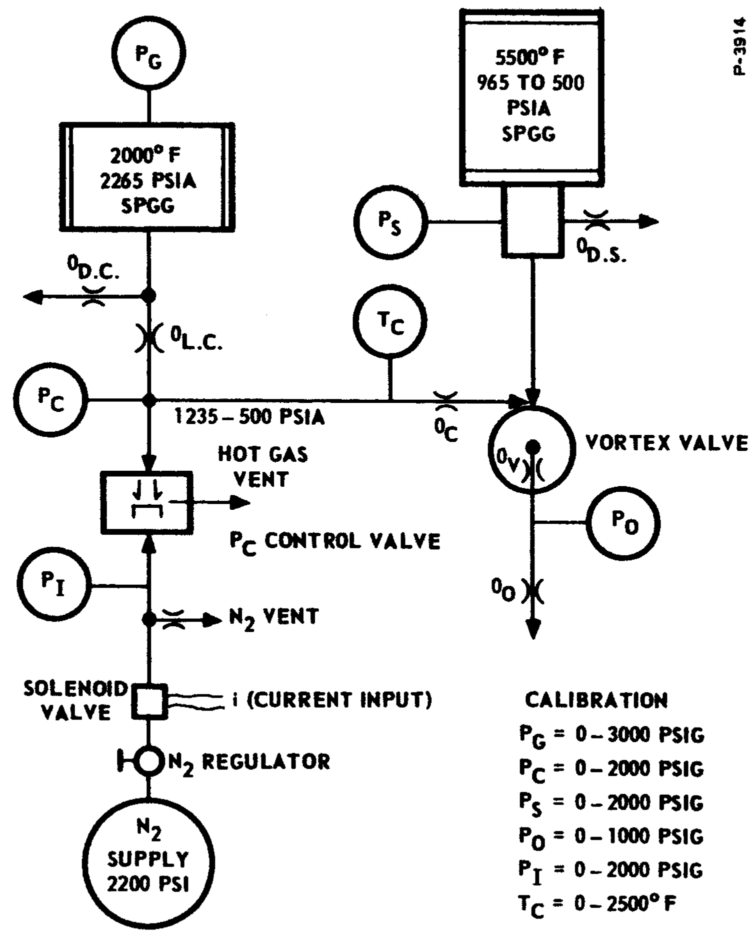


Figure 3 - Test Schematic for #2 Firing of 5500°F SITVC System with Single Vortex Valve

of the recorders, cameras, and timers will be controlled by a sequencer. The time pattern of the sequencer will be as follows:

Time	Function
0 sec	Start high speed camera.
2 sec	Start low speed camera, recorder, timer, and open solenoid valve.
4 sec	Ignite 2000°F SPGG
6 sec	Close solenoid valve.
6.5 sec	Turn off 2000°F SPGG ignitor.
8 sec	Open solenoid valve.

Time	Function
10 sec	Ignite 5500°F SPGG.
11 sec	Close solenoid valve.
14 sec	Open solenoid valve.
16 sec	Close solenoid valve.
18 sec	Open solenoid valve.
20 sec	Close solenoid valve.
22 sec	Open solenoid valve.
24 sec	Close solenoid valve.
26 sec	Open solenoid valve.
28 sec	Close solenoid valve.

The firing procedure for the hot gas test is described in the attached engineering specification. One additional feature added to the firing procedure is the manual override of the 5500°F SPGG ignition in the event that the control system does not function properly in the first portion of the hot test.

PROJECT NO.	THE BENDIX CORPORATION RESEARCH LABORATORIES DIVISION SOUTHFIELD, MICHIGAN	CODE IDENT.	SPECIFICATION NO.	REV.
2830		11272		

ENGINEERING SPECIFICATION

TITLE	DATE
5500°F SITVC System Hot Gas Test Procedure	28 April 1966

Test No. _____ Date ________________

1. General Rules

1.1 There shall be no more than four people in the console room during prefiring checkout.

1.2 After a loaded SPGG has been placed in the test cell, only the following personnel are authorized to enter the cell.

(a) Test site safety engineer.

(b) Assigned test technicians.

(c) Responsible engineer.

1.3 The test cell shall be off limits to all personnel after countdown Item 3.18, except for the following.

(a) Assigned test technician.

(b) Responsible engineer.

2. 2000°F SPGG & Pilot Stage System

2.1 _____All items pertaining to loading of the 2000°F SPGG accomplished as described in Olin test procedure S.O.P. 4549000 and breech assembly print Bendix 2159667.

2.2 _____Couple the 2000°F SPGG to the pilot stage system and torque mounting screws as required.

2.3 _____Install outlet plugs.

2.4 _____Install new burst diaphragm.

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REVISIONS

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2830		11272		

ENGINEERING SPECIFICATION

TITLE	DATE
5500°F SITVC System Hot Gas Test Procedure	28 April 1966

Test No. _____ Date ____________________

3. 5500°F SPGG & SITVC Supply System

3.1 _____All items pertaining to loading accomplished as described in Hercules Powder Test Procedure UOP-066-18-2 for the 5500°F SPGG.

3.2 _____The 5500°F SPGG mounted to the 5500°F SITVC System per Bendix and the mounting screws torqued as required.

3.3 _____Install outlet plugs.

3.4 _____Install new burst diaphragm.

3.5 _____Calibrate pressure pickups and thermocouples using a 4 point calibration. (Leave calibration on strip chart.)

3.6 _____Leak check system mounting surfaces by pressurizing systems to 200 psi.

3.7 _____Remove N_2 supply line and install applicable plug.

3.8 _____Check out sequencer, cameras, recorders, firing circuit.

3.9 _____Run 3 seconds with Camera #1. Show on a place card, the system under going test giving test date, type, and number.

3.10 _____Zero timer.

3.11 _____Install firing cables in shorting plug in console. 2000°F SPGG ____
5500°F SPGG ____

3.12 _____Place firing switch in "OFF" position.

3.13 _____Set recorder speed and timing lines.
(Speed _____ Timing _____)

3.14 _____Set programmer
One complete cycle in _____ seconds)

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ENGINEERING SPECIFICATION

TITLE	DATE
5500°F SITVC System Hot Gas Test Procedure	28 April 1966

Test No. _____ Date _________________

3.15 _____ Notify test site engineer of pending firing.

3.16 _____ Received permission to fire.
(Authorized by _________________)

3.17 _____ Remove outlet plugs from test system.

3.18 _____ Install igniter and igniter cable.

3.19 _____ Open N_2 control source.

3.20 _____ Remove firing cables from shorting plug and in firing position on test panel.

3.21 _____ Turn on D.C. and A.C. power to console.

3.22 _____ Start sequencer and begin one second interval count.

3.23 _____ Observe P_c trace and override 5500°F SPGG ignition if P_c does not modulate in the first seven seconds.

3.24 _____ Notify test site safety engineer firing completed.

3.25 _____ Assign test number and date to strip charts.

Remarks: ____

____

____

____

Test Engineer ______________________

Responsible Engineer ______________________

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